2 Context-Free Languages

(Part 1 of 2)

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2014-05-22

Outline

Context-Free Languages

regular languages 正则语言

- finite automata: DFA / NFA
- regular expressions

some simple languages, such as $\{0^n1^n \mid n \geq 0\}$, are ***not*** regular languages.

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context-free languages 上下文无关语言

- pushdown automata 下推自动机

- first used in the study of human languages
- in the specification and compilation of programming languages
  - ***parser***
  - the construction of a parser from a context-free grammar

Outline

1. Context-Free Grammars
   - Formal Definition of a Context-Free Grammar
   - Examples of a Context-Free Grammar
   - Designing Context-Free Grammars
   - Ambiguity
   - Chomsky Normal Form
2. Pushdown Automata
3. Non-Context-Free Languages

Context-Free Grammars 上下文无关文法

Example (context-free grammar: G_1)

$$A \to 0A1$$

$$A \to B$$

$$B \to \#$$

A grammar

- substitution rules, ***productions*** 产生式
- ***variable*** 变元
- ***terminals*** 终结符
- ***start variable*** 起始变元

Context-Free Grammars 上下文无关文法

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G_1 generates the string $000\#111$

Derivation 推导

- The sequence of substitutions to obtain a string is called a ***derivation***
- $A \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 000A111 \Rightarrow 000B111 \Rightarrow 000\#111$

Context-Free Grammars 上下文无关文法

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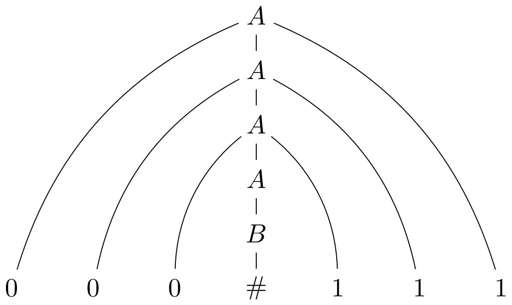

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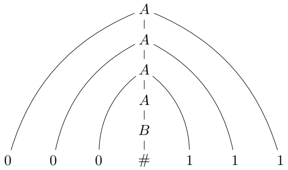

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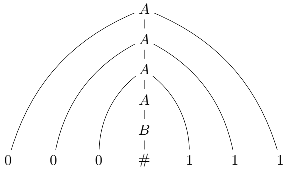

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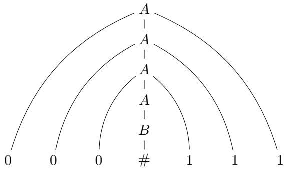

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context-free languages (CFL) 上下文无关语言

- any language that can be generated by some context-free grammar

Abbreviation

$A \Rightarrow 0A1$ and $A \Rightarrow B$

$A \Rightarrow 0A1 \mid B$

Context-Free Grammars 上下文无关文法

Example (context-free grammar G_2)

<SENTENCE> → <NOUN-PHRASE><VERB-PHRASE>

<NOUN-PHRASE> → <CMPLX-NOUN> | <CMPLX-NOUN><PREP-PHRASE>

<VERB-PHRASE> → <CMPLX-VERB> | <CMPLX-VERB><PREP-PHRASE>

<PREP-PHRASE> → <PREP><CMPLX-NOUN>

<CMPLX-NOUN> → <ARTICLE><NOUN>

<CMPLX-VERB> → <VERB> | <VERB><NOUN-PHRASE>

<ARTICLE> → a | the

<NOUN> → boy | girl | flower

<VERB> → touches | likes | sees

<PREP> → with

a boy sees

the boy sees a flower

a girl with a flower likes the boy

Context-Free Grammars 上下文无关文法

Derivation

<SENTENCE> $\Rightarrow$ <NOUN-PHRASE><VERB-PHRASE>
$\Rightarrow$ <CMPLX-NOUN><VERB-PHRASE>
$\Rightarrow$ <ARTICLE><NOUN><VERB-PHRASE>
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$\Rightarrow$ a boy <VERB>
$\Rightarrow$ a boy sees

Formal Definition of a Context-Free Grammar

Definition (CFG (上下文无关文法))

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4. $S \in V$ is the ***start variable***.

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The ***language of the grammar*** is $\{w \in \Sigma^* \mid S \overset{*}{\Rightarrow} w\}$

Formal Definition of a Context-Free Grammar

Example (context-free grammar: G_1)

$G_1 = (V, \Sigma, R, S)$

- $V = \{A, B\}$
- $\Sigma = \{0, 1, \#\}$
- $S = A$
- R:

$$A \to 0A1$$
$$A \to B$$
$$B \to \#$$

Formal Definition of a Context-Free Grammar

Example (context-free grammar: G_3)

$G_3 = (S, a, b, R, S)$

- $$S \to aSb \mid SS \mid \varepsilon$$

Formal Definition of a Context-Free Grammar

Example (context-free grammar: G_4)

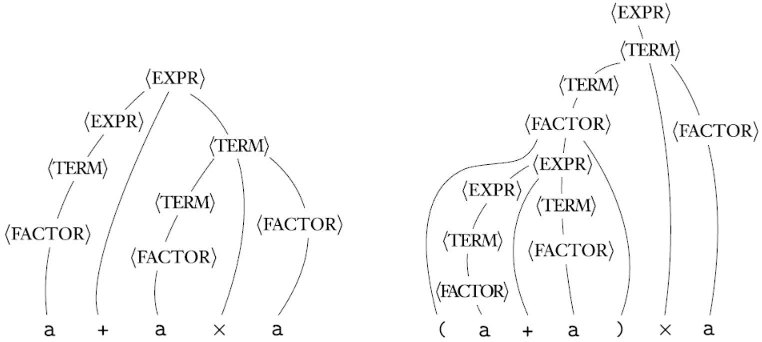
$G_3 = (V, \Sigma, R, <\text{EXPR}>)$

- $V = \{<\text{EXPR}>, <\text{TERM}>, <\text{FACTOR}>\}$
- $\Sigma = \{a, +, \times, (,)\}$
- $$<\text{EXPR}> \to <\text{EXPR}> + <\text{TERM}> | <\text{TERM}>$$
  $$<\text{TERM}> \to <\text{TERM}> \times <\text{FACTOR}> | <\text{FACTOR}>$$
  $$<\text{FACTOR}> \to (<\text{EXPR}>) | a$$

Two strings generated with grammar G_4

- $a + a \times a$
- $(a + a) \times a$

Formal Definition of a Context-Free Grammar



Parse tree for the strings $a + a \times a$ and $(a + a) \times a$

Designing Context-Free Grammars

1. Many CFLs are the union of simpler CFLs

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- $\{0^n 1^n \mid n \geq 0\}$: $S_1 \to 0S_1 1 \mid \varepsilon$
- $\{1^n 0^n \mid n \geq 0\}$: $S_2 \to 1S_2 0 \mid \varepsilon$

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$$S \to S_1 \mid S_2$$
$$S_1 \to 0S_11 \mid \varepsilon$$
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Designing Context-Free Grammars

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Convert any DFA into an equivalent CFG as follows

- Make a variable R_i for each state q_i of the DFA
- Add the rule $R_i \to aR_j$ to the CFG if $\delta(q_i, a) = q_j$ is a transition in the DFA
- Add the rule $R_i \to \varepsilon$ if q_i is an accept state of the DFA
- Make R_0 the start variable of the grammar, where q_0 is the start state of the machine

Designing Context-Free Grammars

3. Certain context-free languages contain strings with two substrings

- $\{0^n 1^n \mid n \geq 0\}$
- $R \rightarrow uRv$

Designing Context-Free Grammars

3. Certain context-free languages contain strings with two substrings
   - $\{0^n 1^n \mid n \geq 0\}$
   - $R \rightarrow uRv$
4. The strings may contain certain structures that appear recursively as part of other (or the same) structures
   - the grammar that generates arithmetic expressions

Ambiguity 二义性

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- If a grammar generates the same string in several different ways, we say that the string is derived ***ambiguously*** in that grammar
- If a grammar generates some string ambiguously, we say that the grammar is ***ambiguous***

Ambiguity

Example (Grammar G_5)

<EXPR>→<EXPR>+<EXPR> | <EXPR>×<EXPR> | (<EXPR>) | a

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This grammar generates the string a+a$\times$a ***ambiguously***

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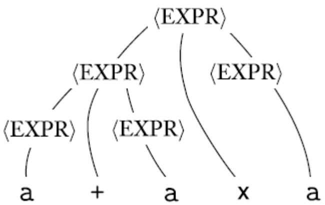

The two parse trees for the string a+a×a in grammar G_5

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Example (Grammar G_5)

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- This grammar doesn't capture the usual precedence relations and so may group the $+$ before the $\times$ or vice versa.
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- A string w is derived ***ambiguously*** in context-free grammar G if it has two or more different leftmost derivations.
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- Grammar G is ***ambiguous*** if it generates some string ambiguously.

Some context-free languages can be generated only by ambiguous grammars. Such languages are called ***inherently ambiguous*** (固有二义性). e.g., $\{a^i b^j c^k \mid i = j \text{ or } j = k\}$

Chomsky Normal Form 乔姆斯基范式

Definition (Chomsky Normal Form (乔姆斯基范式))

A context-free grammar is in ***Chomsky normal form*** if every rule is of the form

$$A \to BC$$
$$B \to a$$

- where a is any terminal and A, B, and C are any variables,
- except that B and C may not be the start variable.
- permit the rule $S \to \varepsilon$, where S is the start variable.

Chomsky Normal Form 乔姆斯基范式

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Any context-free language is generated by a context-free grammar in Chomsky normal form.

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1. add a new start variable
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3. eliminate all ***unit rules*** of the form $A \to B$
4. convert the remaining rules into the proper form

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1. add a new start variable
   - add a new start variable S_0 and the rule $S_0 \rightarrow S$, where S was the original start variable.

Chomsky Normal Form 乔姆斯基范式

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1. add a new start variable
   - add a new start variable S_0 and the rule $S_0 \rightarrow S$, where S was the original start variable.
   - This change guarantees that the start variable doesn't occur on the right-hand side of a rule.

Chomsky Normal Form 乔姆斯基范式

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Proof

2. eliminate all ε-rules of the form $A \to \varepsilon$
    - remove an ε-rule $A \to \varepsilon$, where A is not the start variable.
    - for each occurrence of an A on the right-hand side of a rule, add a new rule with that occurrence deleted.

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   - remove an ε-rule $A \to \varepsilon$, where A is not the start variable.
   - for each occurrence of an A on the right-hand side of a rule, add a new rule with that occurrence deleted.
   - $R \to uAv \Rightarrow R \to uv$
   - $R \to uAvAw \Rightarrow R \to uvAw$, $R \to uAvw$, and $R \to uvw$

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   - If we have the rule $R \to A$, add $R \to \varepsilon$ unless the rule $R \to \varepsilon$ was previously removed.

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   - repeat these steps until we eliminate all ε-rules not involving the start variable

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Proof

3. eliminate all unit rules of the form $A \rightarrow B$

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3. eliminate all unit rules of the form $A \rightarrow B$
   - remove a unit rule $A \rightarrow B$
   - whenever a rule $B \rightarrow u$ appears, we add the rule $A \rightarrow u$ unless this was a unit rule previously removed.

Chomsky Normal Form 乔姆斯基范式

Proof

3. eliminate all unit rules of the form $A \to B$
   - remove a unit rule $A \to B$
   - whenever a rule $B \to u$ appears, we add the rule $A \to u$ unless this was a unit rule previously removed.
   - repeat these steps until we eliminate all unit rules.

Chomsky Normal Form 乔姆斯基范式

Proof

4. convert the remaining rules into the proper form

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4. convert the remaining rules into the proper form
   - replace each rule $A \to u_1 u_2 \cdots u_k$,
     with the rules $A \to u_1 A_1$, $A_1 \to u_2 A_2$, $A_2 \to u_3 A_3$, $\cdots$, and
     $A_{k-2} \to u_{k-1} u_k$
     - where $k \geq 3$
     - each u_i is a variable or terminal symbol
     - A_i's are new variables

Chomsky Normal Form 乔姆斯基范式

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4. convert the remaining rules into the proper form
   - replace each rule $A \to u_1 u_2 \cdots u_k$,
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     $A_{k-2} \to u_{k-1} u_k$
     - where $k \geq 3$
     - each u_i is a variable or terminal symbol
     - A_i's are new variables
   - replace any terminal u_i in the preceding rule(s) with the new variable U_i and add the rule $U_i \to u_i$

Chomsky Normal Form

Example

Let G_6 be the following CFG and convert it to Chomsky normal form by using the conversion procedure just given.

1. The original CFG G_6 is shown on the left. The result of applying the first step to make a new start variable appears on the right.

$$S \to ASA \mid aB$$
$$A \to B \mid S$$
$$B \to b \mid \varepsilon$$

Chomsky Normal Form

Example

Let G_6 be the following CFG and convert it to Chomsky normal form by using the conversion procedure just given.

1. The original CFG G_6 is shown on the left. The result of applying the first step to make a new start variable appears on the right.

$$S \to ASA \mid aB$$
$$A \to B \mid S$$
$$B \to b \mid \varepsilon$$

$$S_0 \to S$$
$$S \to ASA \mid aB$$
$$A \to B \mid S$$
$$B \to b \mid \varepsilon$$

Chomsky Normal Form

Example

Let G_6 be the following CFG and convert it to Chomsky normal form by using the conversion procedure just given.

2. Remove ε-rules $B \to \varepsilon$, shown on the left, and $A \to \varepsilon$, shown on the right.

$$S_0 \to S$$
$$S \to ASA \mid aB \mid a$$
$$A \to B \mid S \mid \varepsilon$$
$$B \to b \mid \varepsilon$$

Chomsky Normal Form

Example

Let G_6 be the following CFG and convert it to Chomsky normal form by using the conversion procedure just given.

2. Remove ε-rules $B \to \varepsilon$, shown on the left, and $A \to \varepsilon$, shown on the right.

$S_0 \to S$	$S_0 \to S$
$S \to ASA \mid aB \mid a$	$S \to ASA \mid aB \mid a \mid SA \mid AS \mid S$
$A \to B \mid S \mid \varepsilon$	$A \to B \mid S \mid \varepsilon$
$B \to b \mid \varepsilon$	$B \to b$

Chomsky Normal Form

Example

Let G_6 be the following CFG and convert it to Chomsky normal form by using the conversion procedure just given.

3. (a) Remove unit rules $S \to S$, shown on the left, and $S_0 \to S$, shown on the right.

$S_0 \to S$

$S \to ASA \mid aB \mid a \mid SA \mid AS \mid S$

$A \to B \mid S$

$B \to b$

Chomsky Normal Form

Example

Let G_6 be the following CFG and convert it to Chomsky normal form by using the conversion procedure just given.

3. (a) Remove unit rules $S \to S$, shown on the left, and $S_0 \to S$, shown on the right.

$S_0 \to S$	$S_0 \to S \mid ASA \mid aB \mid a \mid SA \mid AS$
$S \to ASA \mid aB \mid a \mid SA \mid AS \mid S$	$S \to ASA \mid aB \mid a \mid SA \mid AS$
$A \to B \mid S$	$A \to B \mid S$
$B \to b$	$B \to b$

Chomsky Normal Form

Example

Let G_6 be the following CFG and convert it to Chomsky normal form by using the conversion procedure just given.

3. (b) Remove unit rules $A \to B$ and $A \to S$.

$S_0 \to ASA \mid aB \mid a \mid SA \mid AS$

$S \to ASA \mid aB \mid a \mid SA \mid AS$

$A \to B \mid S \mid b$

$B \to b$

Chomsky Normal Form

Example

Let G_6 be the following CFG and convert it to Chomsky normal form by using the conversion procedure just given.

3. (b) Remove unit rules $A \to B$ and $A \to S$.

$$S_0 \to ASA \mid aB \mid a \mid SA \mid AS$$
$$S \to ASA \mid aB \mid a \mid SA \mid AS$$
$$A \to B \mid S \mid b$$
$$B \to b$$

$$S_0 \to ASA \mid aB \mid a \mid SA \mid AS$$
$$S \to ASA \mid aB \mid a \mid SA \mid AS$$
$$A \to S \mid b \mid ASA \mid aB \mid a \mid SA \mid AS$$
$$B \to b$$

Chomsky Normal Form

Example

Let G_6 be the following CFG and convert it to Chomsky normal form by using the conversion procedure just given.

4. Convert the remaining rules into the proper form by adding additional variables and rules. The final grammar in Chomsky normal formis equivalent to G_6.

$$S_0 \to AA_1 \mid UB \mid a \mid SA \mid AS$$
$$S \to AA_1 \mid UB \mid a \mid SA \mid AS$$
$$A \to b \mid AA_1 \mid UB \mid a \mid SA \mid AS$$
$$A_1 \to SA$$
$$U \to a$$
$$B \to b$$

Outline

1. Context-Free Grammars

2. Pushdown Automata
   - Formal Definition of a Pushdown Automaton
   - Examples of Pushdown Automata

3. Non-Context-Free Languages

Outline