1 Regular Languages

(Part 1 of 2)

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Outline

A Computational Model

The theory of computation begins with a question:

What is a computer?

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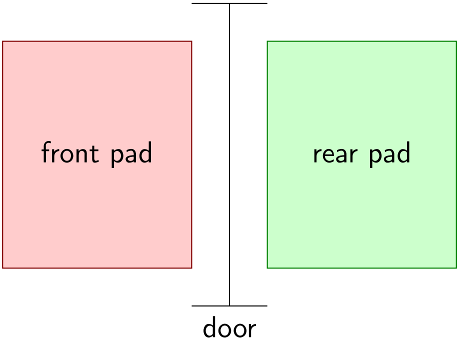
- ***Computational model***: an idealized computer.
- Several different computational models
  - ***Finite automata*** or ***finite state machine*** 有穷自动机
  - ***Pushdown automata*** 下推自动机
  - ***Linear-bounded automata*** 线性有界自动机
  - ***Turing machine*** 图灵机

Outline

Example: An Automatic Door



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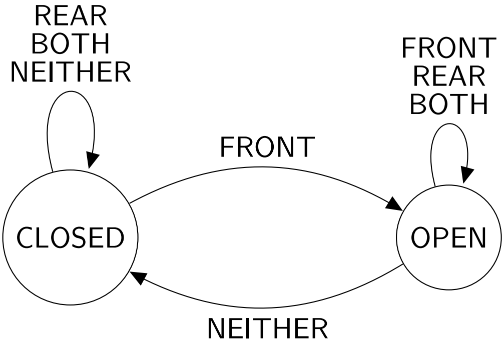

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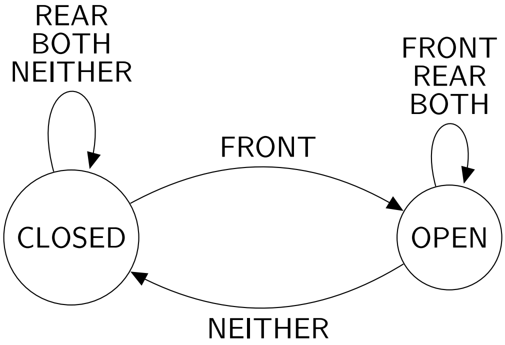
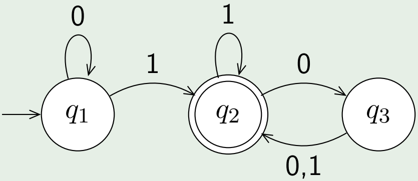


inpute signal

state		NEITHER	FRONT	REAR	BOTH
	CLOSED	CLOSED	OPEN	CLOSED	CLOSED
	OPEN	CLOSED	OPEN	OPEN	OPEN

Example: A Finite Automaton

Example (A finite automaton M_1)



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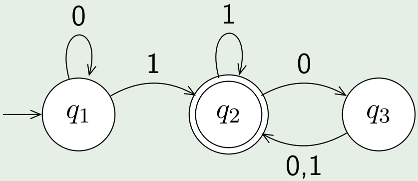

- the ***state diagram*** of M_1
- three ***states***: q_1, q_2, and q_3
- the ***start state***: q_1
- the ***accept state***: q_2
- ***transitions***: the arrows going from one state to another

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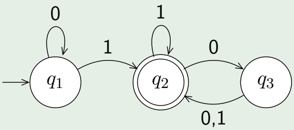

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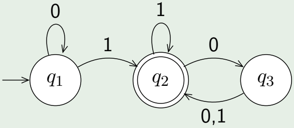

Feed the input string 1101 to the machine M_1

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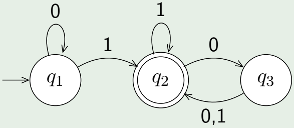

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1. Start in state q_1

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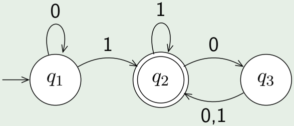

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1. Start in state q_1
2. Read 1, follow transition from q_1 to q_2

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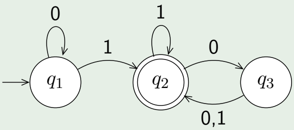

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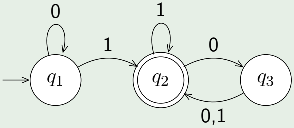

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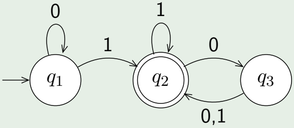

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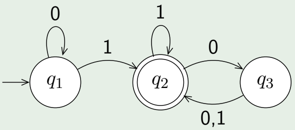

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6. ***Accept*** because M_1 is in an accept state q_2 at the end of the input

Formal Definition of a Finite Automaton

Definition (DFA (确定型有穷自动机))

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A ***deterministic finite automaton*** (DFA) is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

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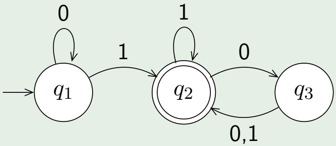
5. $F \subseteq Q$ is the ***set of accept states***.

Using the Definition of DFA

Example (A finite automaton M_1)



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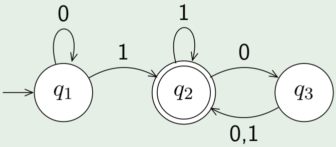

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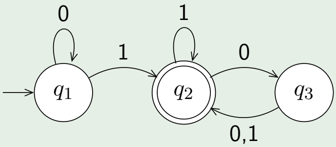

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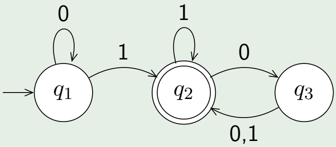

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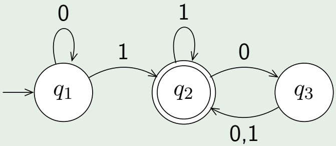

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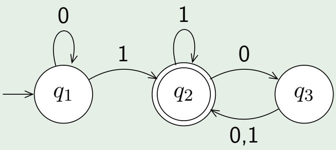

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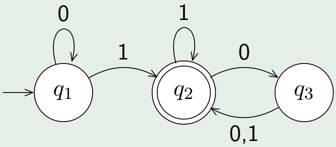

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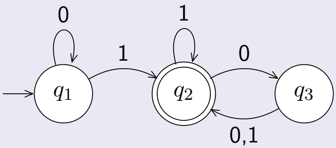
5. $F = \{q_2\}$

Language of DFA



- If A is the set of all strings that machine M accepts, we say that A is the ***language of machine*** M and write $L(M) = A$.

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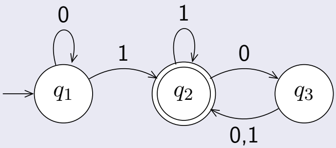

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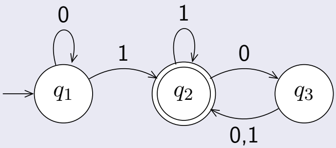

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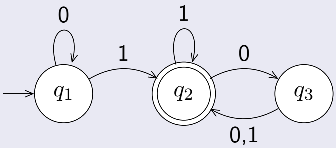

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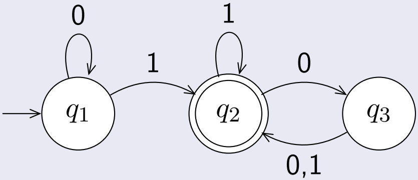
- A machine may accept several strings, but it always recognizes only one language.
- What about the machine accepts no strings?

Language of DFA M_1

DFA M_1



$L(M_1) =$?

Language of DFA M_1

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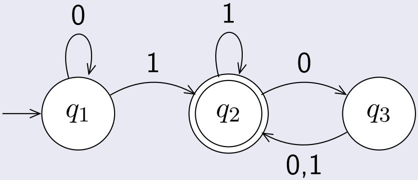


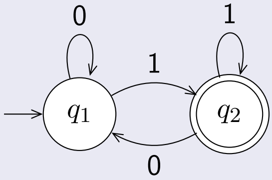
$L(M_1) = ?$

$L(M_1) =$

$$A = \{w \mid w \text{ contains at least one 1 and an even number of 0s follow the last 1 }\}$$

Example: DFA M_2

DFA M_2



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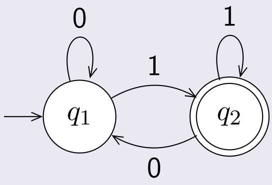

$$M_2 = (\{q_1, q_2\}, \{0, 1\}, \delta, q_1, \{q_2\})$$

δ:

	0	1
q_1	q_1	q_2
q_2	q_1	q_2

Example: DFA M_2

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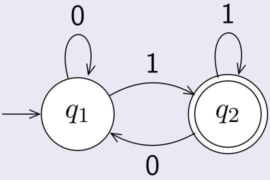

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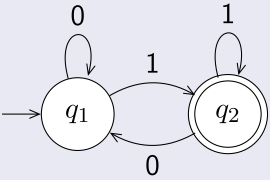

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try 1101,

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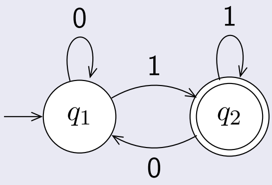

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try 1101, try 110

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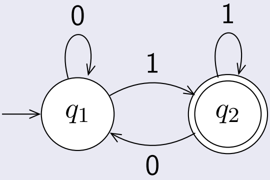

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try 1101, try 110

$$L(M_2) = \{w \mid w \text{ ends in a } 1\}$$

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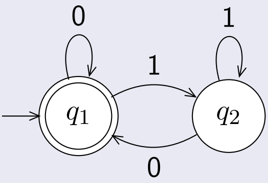
DFA M_3



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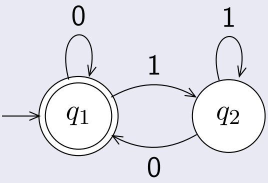

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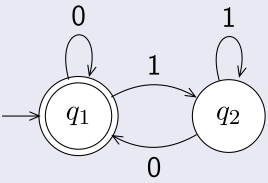
DFA M_3



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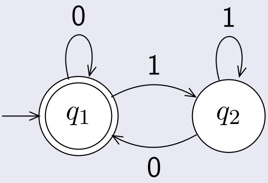
$L(M_3) =?$

$L(M_3) = \{w \mid w$ is the empty string ε or ends in a 0$\}$

Example: DFA M_3

DFA M_3



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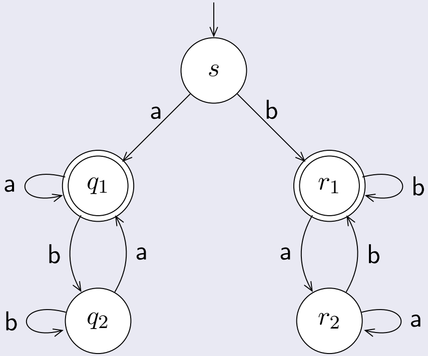
$L(M_3) =$?

$$L(M_3) = \{w \mid w \text{ is the empty string } \varepsilon \text{ or ends in a } 0\}$$

What is the relationship between $L(M_2)$ and $L(M_3)$?

Example: DFA M_4

DFA M_4



Example: DFA M_4

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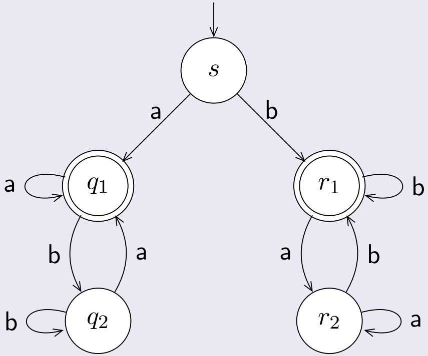

$L(M_4) =$

Example: DFA M_4

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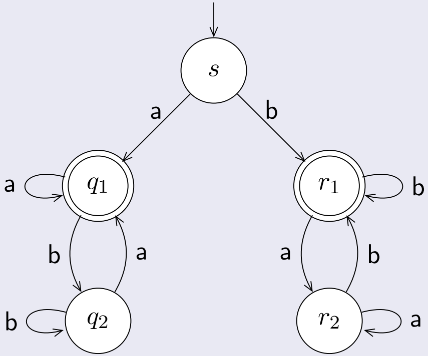
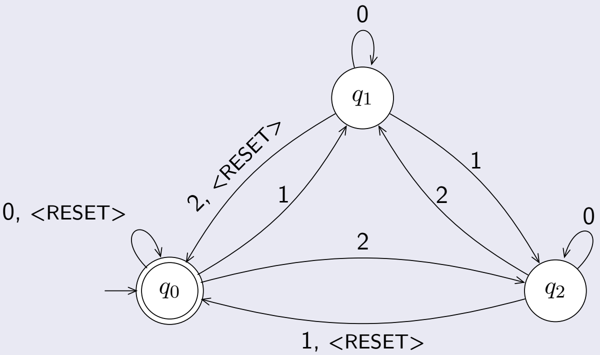


$L(M_4) = \{w \mid w$ starts and ends with the same symbol $\}$

Example: DFA M_5

DFA M_5



Example: DFA M_5

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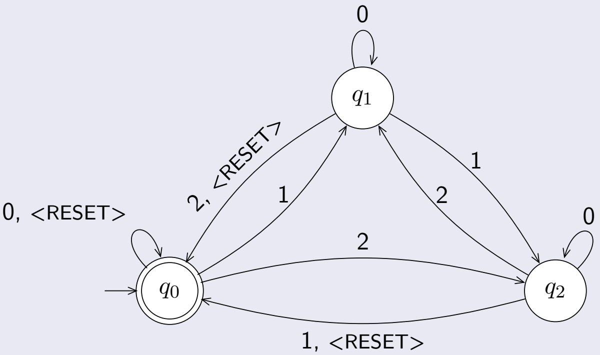

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    $\delta_i(q_j, 2) = q_k$, where $k = j + 2$ modulo i

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Formal Definition of Computation for a DFA

- Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA.
- Let $w = a_1 a_2 \dots a_n$ be a string where $a_i \in \Sigma$.
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We say that M recognizes language A if $A = \{w \mid M \text{ accepts } w\}$

Regular Language

Definition (regular language)

A language is called a regular language if some DFA recognizes it.

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$L(M_5) = \{w \mid$ the sum of the symbols in w is 0 modulo 3, except that <RESET> resets the count to 0 $\}$

Designing Finite Automata

An approach helpful: "reader as automaton"

- put ***yourself*** in the place of the machine you are trying to design
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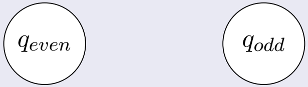

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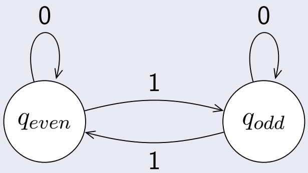

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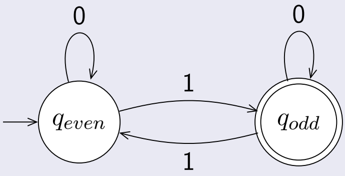

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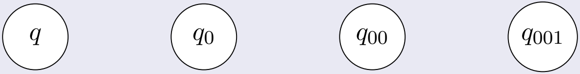

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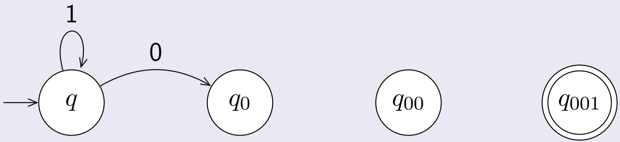

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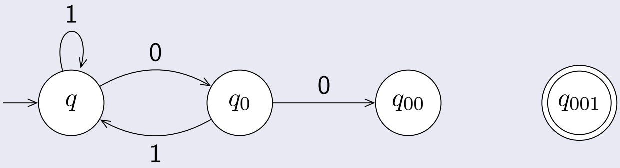

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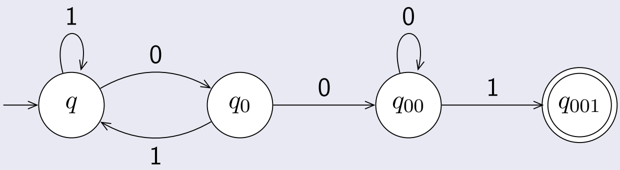

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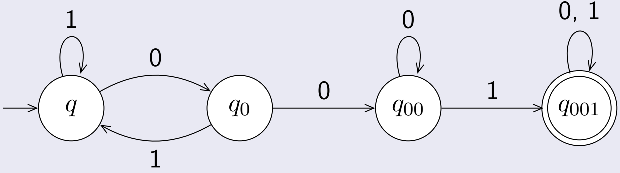

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2. Σ is the same as in M_1 and M_2

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5. $F = \{(r_1, r_2) \mid r_1 \in F_1$ or $r_2 \in F_2\}$

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Problem: M doesn't know where to break the input string?

Outline

Nondeterminism 非确定性

- ***Determinism***: When the machine is in a given state and reads the next input symbol, we know what the next state will be it is determined. We call this ***deterministic*** computation.
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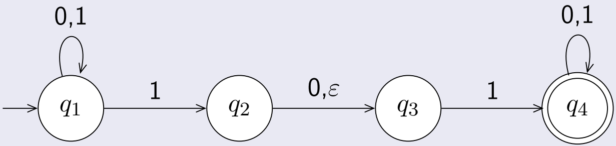
- Nondeterminism is a generalization of determinism,
- so every deterministic finite automaton is automatically a nondeterministic finite automaton.

Nondeterministic Finite Automata

NFA N_1

- Nondeterministic finite automata may have additional features.



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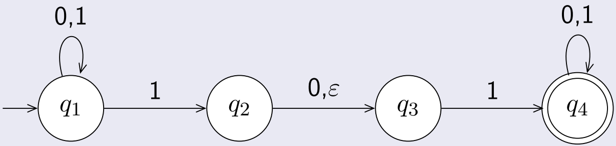
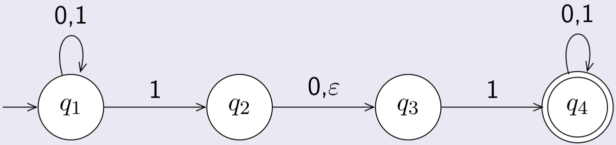


- DFA: deterministic finite automaton 确定型有穷自动机
- NFA: nondeterministic finite automaton 非确定型有穷自动机

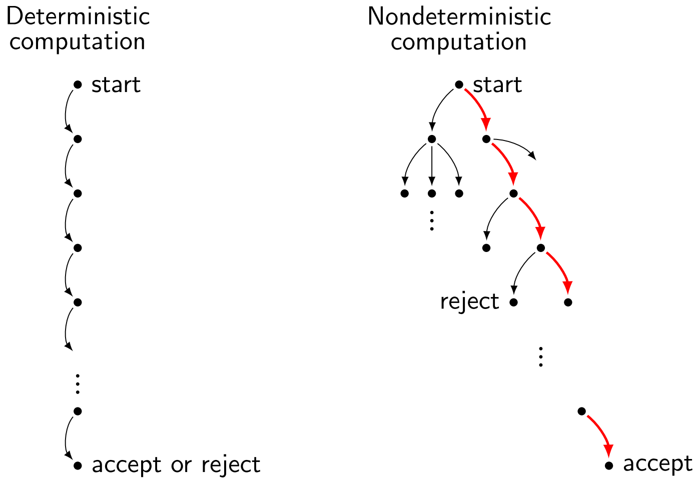
NFAs

NFA N_1

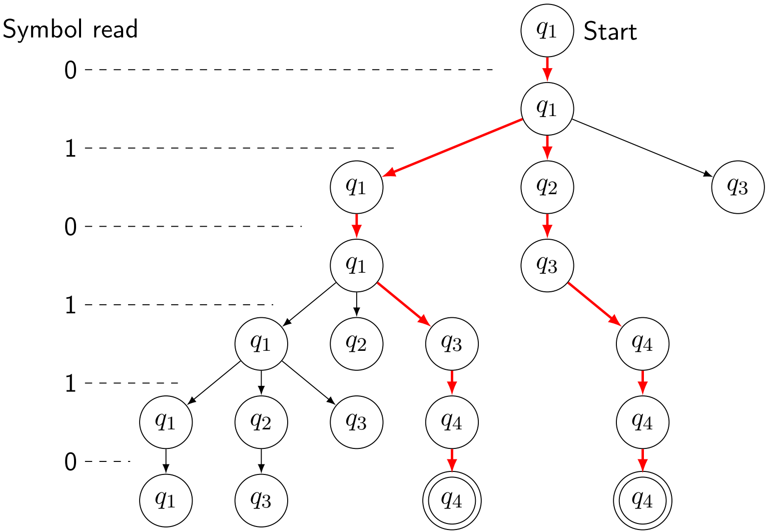


- DFA:
  1. every state of a DFA always has exactly one exiting transition arrow for each symbol in the alphabet.
- NFA:
  1. a state may have zero, one, or many exiting arrows for each alphabet symbol.
  2. an NFA may have arrows labeled with members of the alphabet or ε.

Deterministic and Nondeterministic Computations



How Does an NFA Compute?



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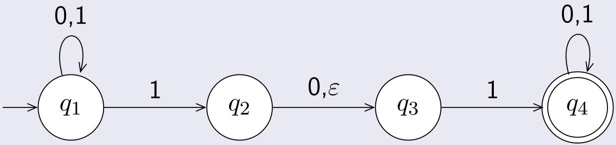

- $L(N_1) = ?$

NFAs

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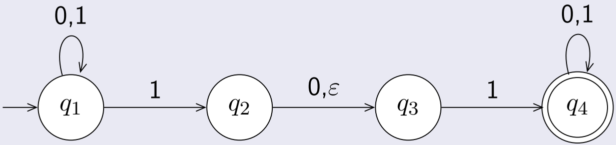

- $L(N_1) = ?$ $\{w \mid w$ contain either 101 or 11 as a substring$\}$

Example: NFA N_2

- The language A:
  - $\{$the language consisting of all strings over $\{0, 1\}$ containing a 1 in the third position from the end$\}$
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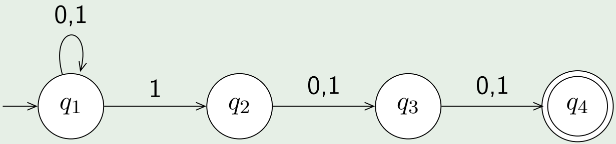

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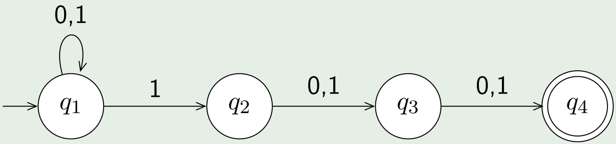
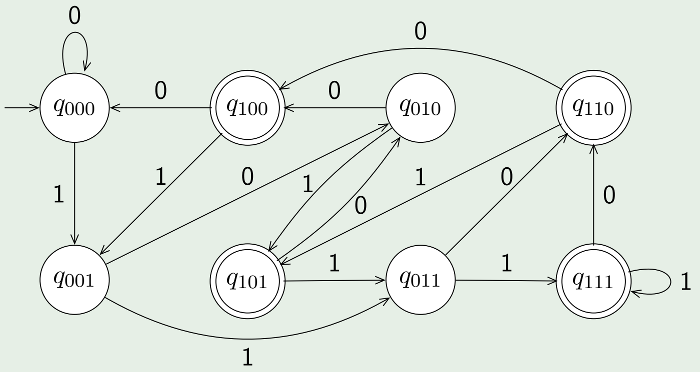


$L(N_2) = A$

Example: NFA N_2

Every NFA can be converted into an equivalent DFA.

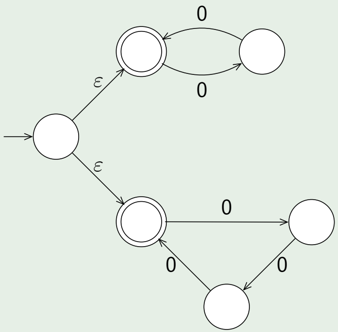
Example (The equivalent DFA of NFA N_2)



Example: NFA N_3

The convenience of having ε arrows

Example (NFA N_3)



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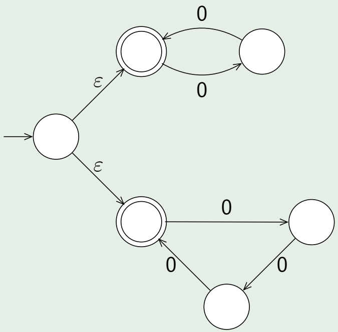
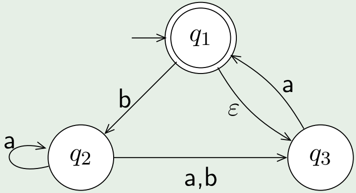


$L(N_3) = \{$all strings of the form 0^k where k is a multiple of 2 or 3. $\}$

Example: NFA N_4

Example (NFA N_4)



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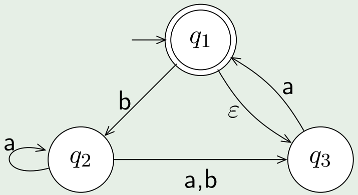


- it accepts the strings ε, a, baba, baa
- it accepts it doesn't accept the strings b, bb, babba

Formal Definition of a Nondeterministic Finite Automaton

Definition (NFA)

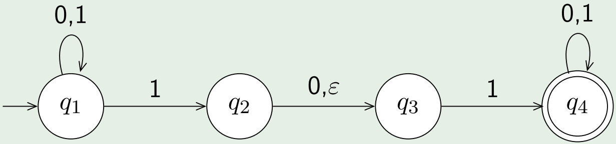
A ***nondeterministic finite automaton*** (NFA) is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set of states,
2. Σ is a finite alphabet,
3. $\delta : Q \times \Sigma_\varepsilon \to \mathcal{P}(Q)$ is the transition function,
4. $q_0 \in Q$ is the start state, and
5. $F \subseteq Q$ is the set of accept states.

- $\mathcal{P}(Q)$ is the power set of Q
- $\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$

Example: The Formal Definition of NFA N_1

Example (Recall the NFA N_1)



$N_1 = (Q, \Sigma, \delta, q_1, F)$, where

- $Q = \{q_1, q_2, q_3, q_4\}$
- $\Sigma = \{0, 1\}$
- δ is given as
- q_1 is the start state
- $F = \{q_4\}$

	0	1	ε
q_1	$\{q_1\}$	$\{q_1, q_2\}$	$\emptyset$
q_2	$\{q_3\}$	$\emptyset$	$\{q_3\}$
q_3	$\emptyset$	$\{q_4\}$	$\emptyset$
q_4	$\{q_4\}$	$\{q_4\}$	$\emptyset$

Formal Definition of Computation for an NFA

- Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA.
- Let w be a string over Σ.
- Then N accepts w if we can write w as $w = a_1 a_2 \cdots a_n$, where $a_i \in \Sigma_\varepsilon$ and a sequence of states $r_0, r_1, \ldots, r_n$ exists in Q with three conditions:
  1. $r_0 = q_0$
  2. $r_{i+1} \in \delta(r_i, a_{i+1})$, for $i = 0, \ldots, n-1$
  3. $r_n \in F$

Equivalence of NFAs and DFAs

DFA and NFA recognize the same class of languages.

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- Useful: describing an NFA for a given language sometimes is much easier than describing a DFA for that language

Equivalent

Say that two machines are ***equivalent*** if they recognize the same language.

Equivalence of NFAs and DFAs

Theorem

Every nondeterministic finite automaton has an equivalent deterministic finite automaton.

Equivalence of NFAs and DFAs

Theorem

Every nondeterministic finite automaton has an equivalent deterministic finite automaton.

Proof.

- Let $N = (Q, \Sigma, \delta, q_0, F)$ be the NFA recognizing some language A.
- We construct a DFA $M = (Q', \Sigma, \delta', q_0', F')$ recognizing A.
- Before doing the full construction, let's first consider the easier case wherein N has no ε arrows. Later we take the ε arrows into account.

□

Equivalence of NFAs and DFAs

Theorem

Every nondeterministic finite automaton has an equivalent deterministic finite automaton.

Proof.

1. $Q' = \mathcal{P}(Q)$
2. For $R \in Q'$ and $a \in \Sigma$,

   $\delta'(R, a) = \{q \in Q \mid q \in \delta(r, a)$ for some $r \in R\}$

$$\delta'(R, a) = \bigcup_{r \in R} \delta(r, a)$$

□

Equivalence of NFAs and DFAs

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Every nondeterministic finite automaton has an equivalent deterministic finite automaton.

Proof.

3. $q'_0 = \{q_0\}$
4. $F' = \{R \in Q' \mid R \text{ contains an accept state of } N\}$

□

Equivalence of NFAs and DFAs

Proof.

Now we need to consider the ε arrows.

Equivalence of NFAs and DFAs

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Now we need to consider the ε arrows.

- For any state R of M,
  $E(R) = \{q \mid q$ can be reached from R by traveling along 0 or more ε arrows$\}$
  - $E(R)$ is the collection of states that can be reached from members of R by going only along ε arrows, including the members of R themselves.
- $\delta'(R, a) = \{q \in Q \mid q \in E(\delta(r, a))$ for some $r \in R\}$
- $q'_0 = E(\{q_0\})$

Equivalence of NFAs and DFAs

Proof.

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- $\delta'(R, a) = \{q \in Q \mid q \in E(\delta(r, a))$ for some $r \in R\}$
- $q_0' = E(\{q_0\})$

We have now completed the construction of the DFA M that simulates the NFA N. □

Equivalence of NFAs and DFAs

Theorem

Every nondeterministic finite automaton has an equivalent deterministic finite automaton.

Corollary

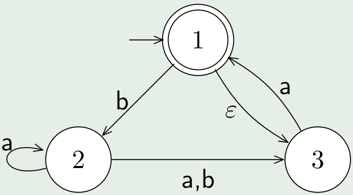
A language is regular if and only if some nondeterministic finite automaton recognizes it.

Equivalence of NFAs and DFAs

Example (NFA N_4)

NFA $N_4 = (Q, \Sigma, \delta, q_0, F)$

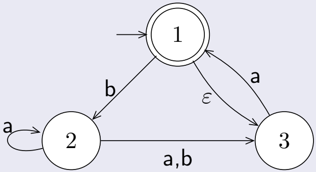
- $Q = \{1, 2, 3\}$
- $\Sigma = \{\text{a, b}\}$
- δ
- $q_0 = 1$
- $F = \{1\}$



Construct a DFA D that is equivalent to N_4

Equivalence of NFAs and DFAs

NFA $N_4 = (Q, \Sigma, \delta, q_0, F)$



DFA $D = (Q', \Sigma, \delta', q_0', F')$

Equivalence of NFAs and DFAs

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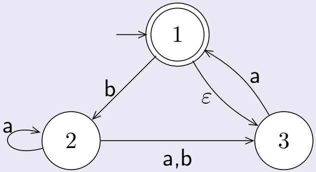

DFA $D = (Q', \Sigma, \delta', q_0', F')$

- $Q' = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Equivalence of NFAs and DFAs

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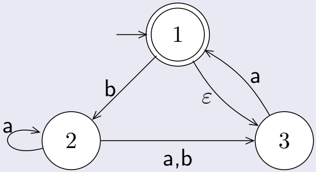

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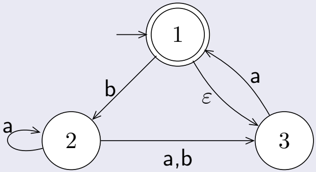

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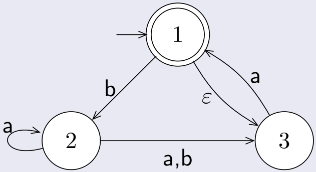

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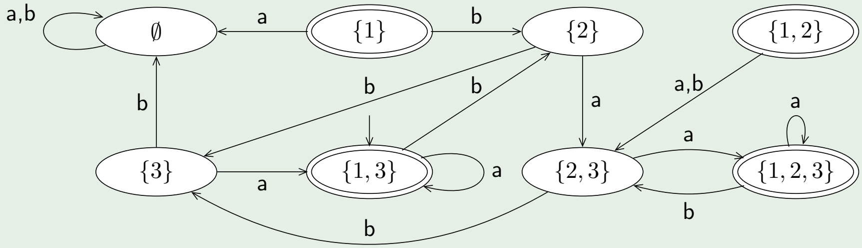
- $F' = \{\{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}$

Equivalence of NFAs and DFAs

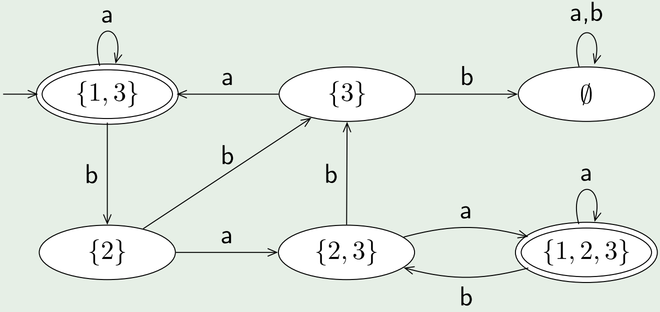
Example (DFA D that is equivalent to the NFA N_4)



Equivalence of NFAs and DFAs

Example (DFA D after removing unnecessary states)

- No arrows point at states {1} and {1, 2}
- They may be removed without affecting the performance of DFA.



Closure Under the Regular Operations

Theorem

The class of regular languages is closed under the union operation.

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Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize $A_1 \cup A_2$.

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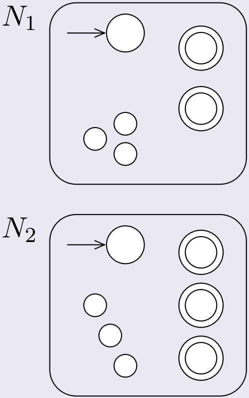
$$\delta(q, a) = \begin{cases} \delta_1(q, a) & q \in Q_1 \\ \delta_2(q, a) & q \in Q_2 \\ \{q_1, q_2\} & q = q_0 \text{ and } a = \varepsilon \\ \emptyset & q = q_0 \text{ and } a \neq \varepsilon \end{cases}$$

□

Closure Under the Regular Operations

Construction of an NFA N to recognize $A_1 \cup A_2$



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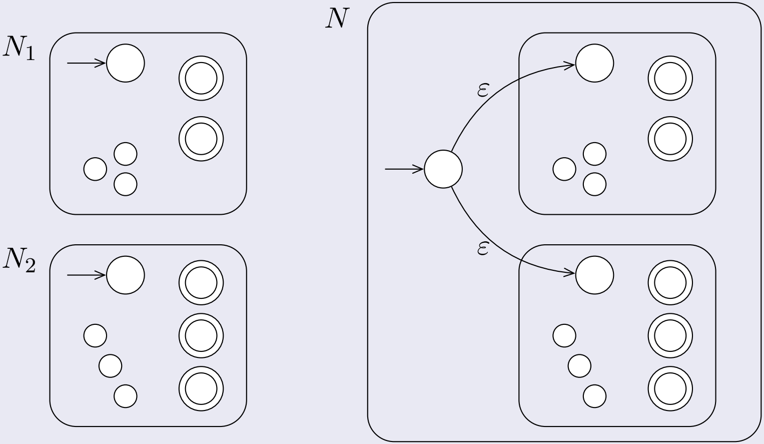

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1. $Q = Q_1 \cup Q_2$
2. q_1 is the same as the start state of N_1

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3. The accept states F_2 are the same as the accept states of N_2

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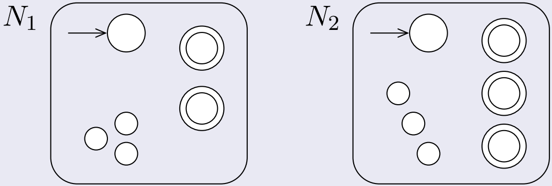
4. For any $q \in Q$ and any $a \in \Sigma_\varepsilon$

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \text{ and } q \notin F_1 \\ \delta_1(q,a) & q \in F_1 \text{ and } a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_2\} & q \in F_1 \text{ and } a = \varepsilon \\ \delta_2(q,a) & q \in Q_2 \end{cases}$$

□

Closure Under the Regular Operations

Construction of N to recognize $A_1 \circ A_2$



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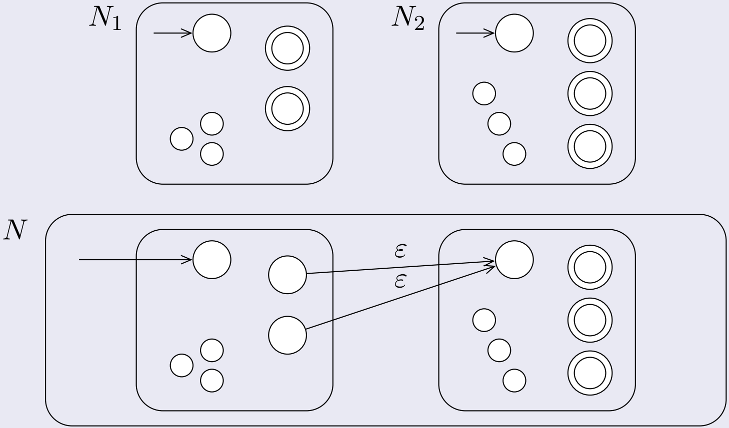

Closure Under the Regular Operations

Theorem

The class of regular languages is closed under the star operation.

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Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1.

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Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1.

Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize A_1^*.

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1. $Q = \{q_0\} \cup Q_1$

□

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1. $Q = \{q_0\} \cup Q_1$
2. q_0 is the new start state.

□

Closure Under the Regular Operations

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The class of regular languages is closed under the star operation.

Proof.

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1.

Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize A_1^*.

1. $Q = \{q_0\} \cup Q_1$
2. q_0 is the new start state.
3. $F = \{q_0\} \cup F_1$

□

Closure Under the Regular Operations

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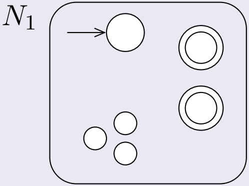
$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \text{ and } q \notin F_1 \\ \delta_1(q,a) & q \in F_1 \text{ and } a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_1\} & q \in F_1 \text{ and } a = \varepsilon \\ \{q_1\} & q = q_0 \text{ and } a = \varepsilon \\ \emptyset & q = q_0 \text{ and } a \neq \varepsilon \end{cases}$$

□

Closure Under the Regular Operations

Construction of N to recognize A_1^*



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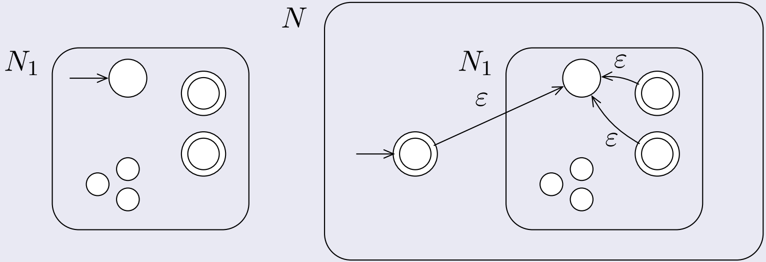

Conclusion

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  - Equivalence of NFAs and DFAs
  - Closure Under the Regular Operations