Chapter 9 Morphological Image Processing

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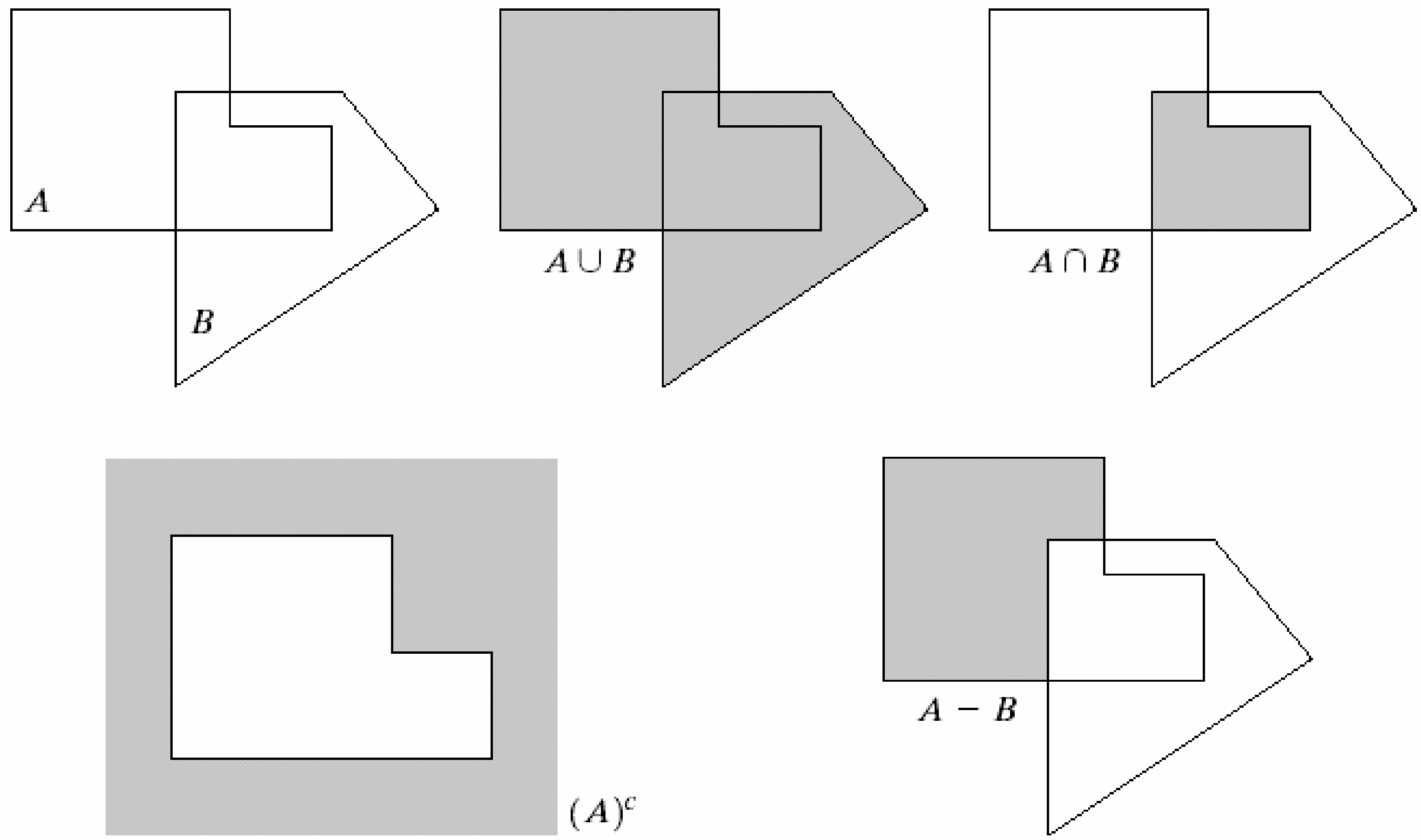
- The material in this chapter begins a transition from a focus on purely image processing methods whose input and output are images, to processes in which

  the inputs are images, but the outputs are attributes extracted from those images.

- Preliminaries
- Dilation and Erosion
- Opening and Closing
- The Hit-or-Miss Transformation
- Some Basic Morphological Algorithms
- Extensions to Gray-Scale Images

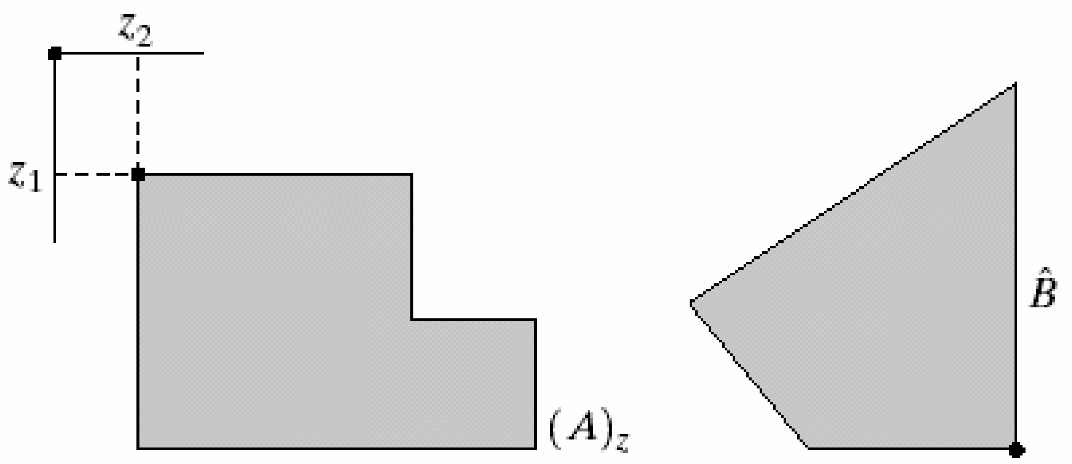
- Some Basic Concepts from Set Theory
- Logic Operations Involving Binary Images

- Union: $C = A \bigcup B$
- Intersection: $D = A \bigcap B$
- Disjoint: $A \bigcap B = \Phi$
- Complement: $A^c = \{ w \mid w \notin A \}$
- Difference: $A - B = \{ w \mid w \in A, w \notin B \} = A \bigcap B^c$
- Reflection: $\hat{B} = \{ w \mid w = -b, \quad for \quad b \in B \}$
- Translation: $(A)_z = \{ c = a + z, \quad for \quad a \in A \}$



a b c
d e

FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.



a b

FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.

TABLE 9.1
The three basic logical operations.

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

- Some Basic Concepts from Set Theory
- Logic Operations Involving Binary Images

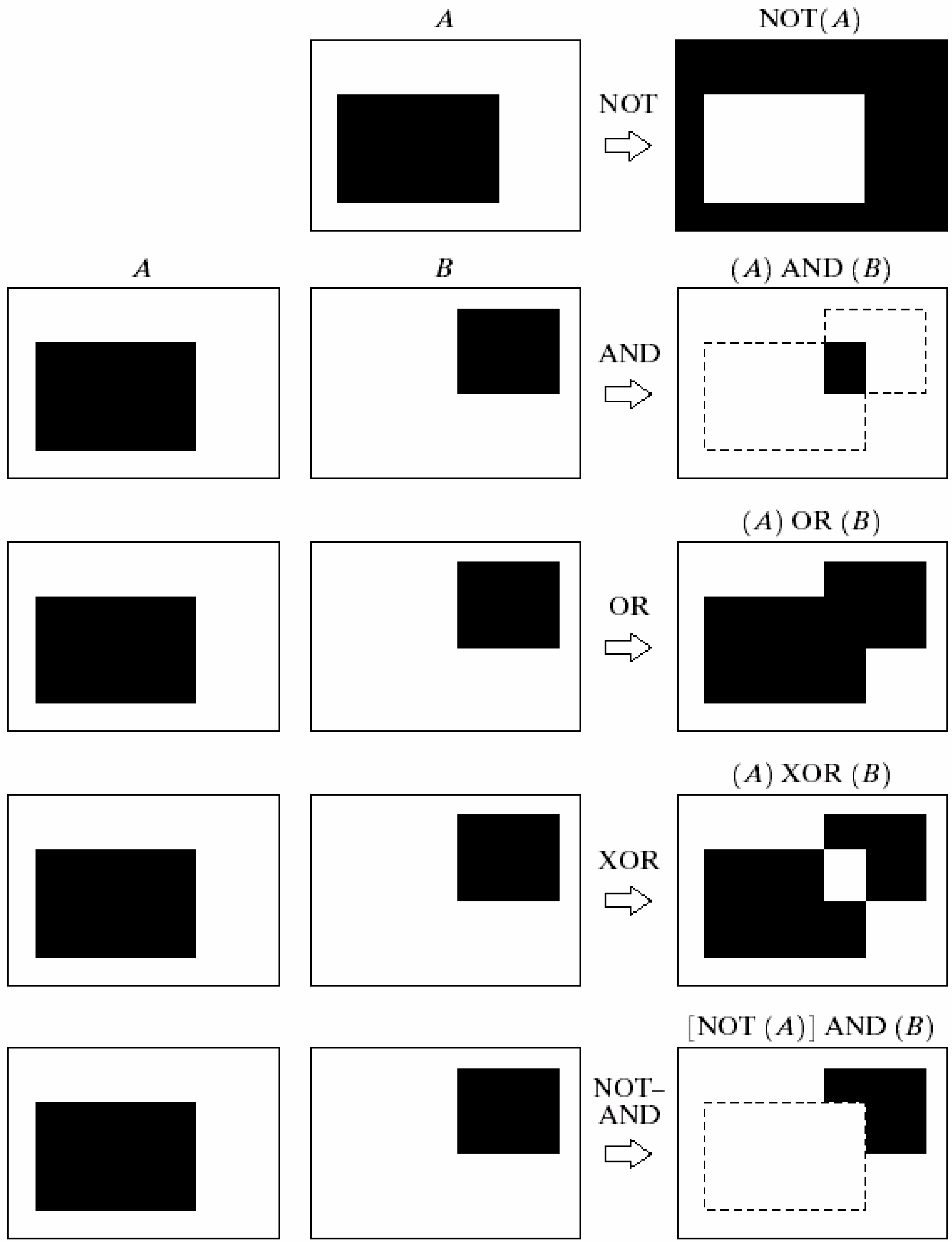


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

- Preliminaries
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- Dilation
- Erosion

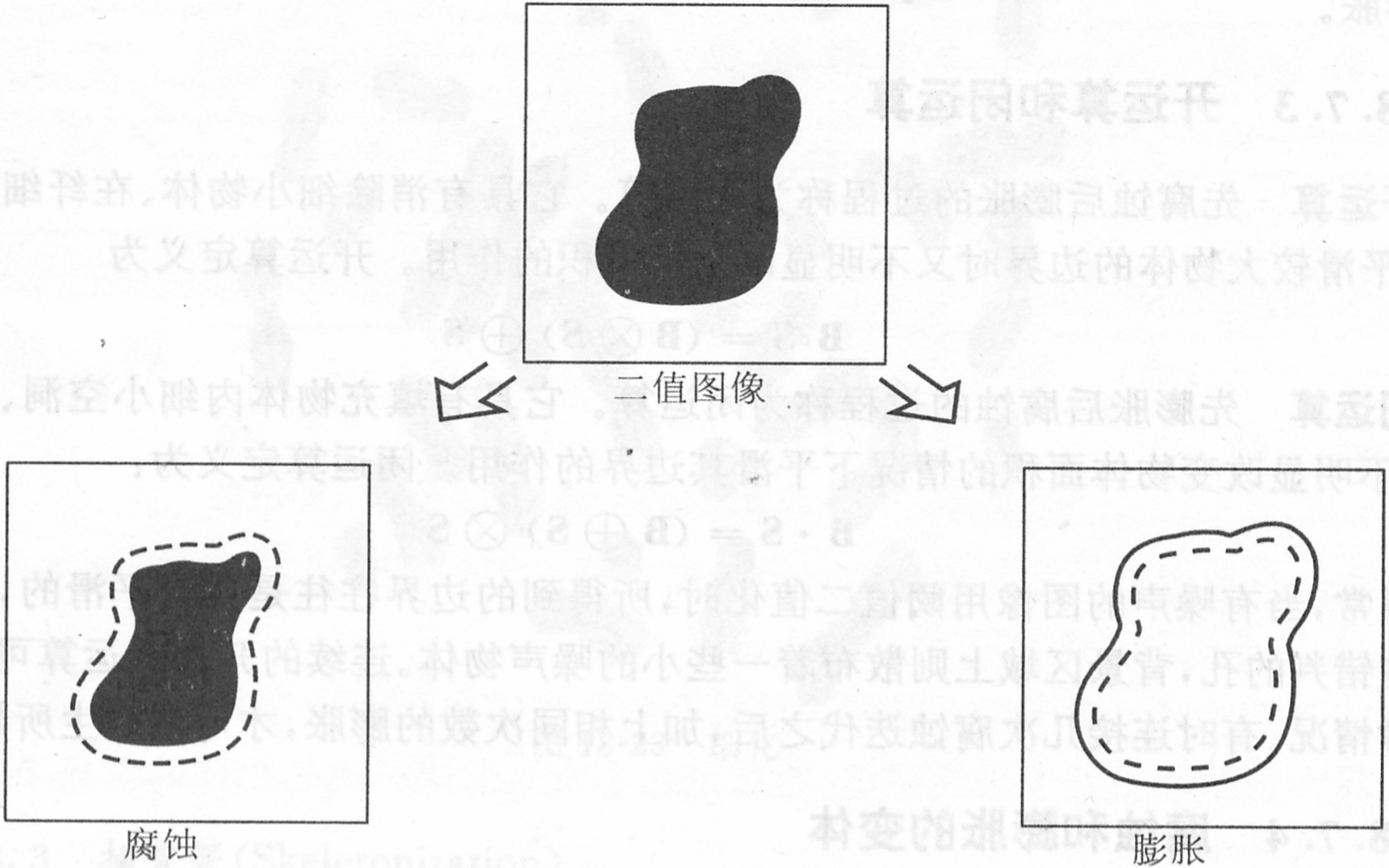


图 18-22 腐蚀和膨胀

- With A and B as sets in Z^2, the dilation of A by B, denoted $A \oplus B$, is defined as

$$A \oplus B = \left\{ z \mid \left(\hat{B}\right)_z \cap A \neq \Phi \right\}$$

or

$$A \oplus B = \left\{ z \mid \left[\left(\hat{B}\right)_z \cap A\right] \subseteq A \right\}$$

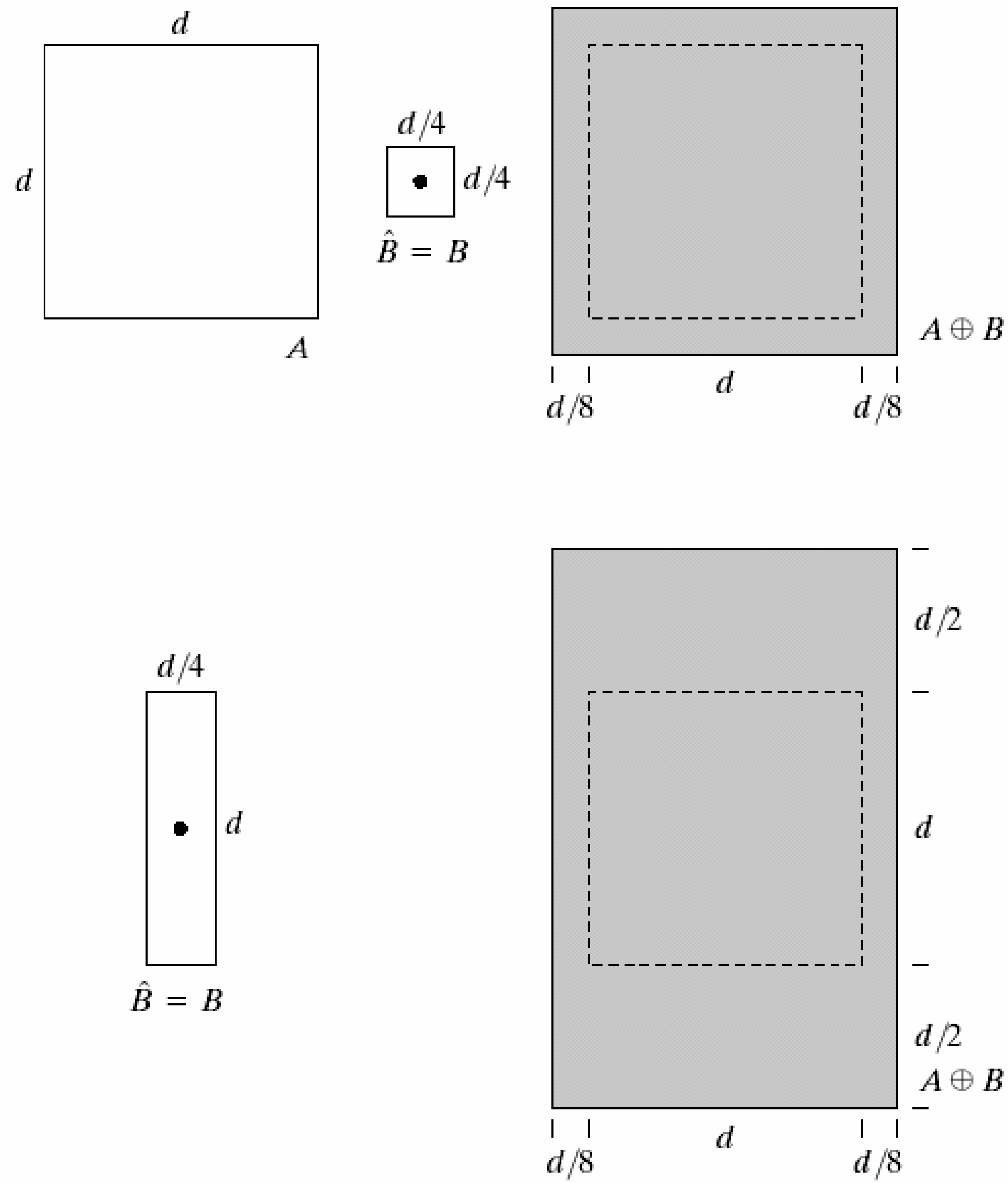


FIGURE 9.4
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.

- Dilation
- Erosion

- For sets A and B in Z^2, the erosion of A by B, denoted

$$A \Theta B = \left\{ z \mid \left(B \right)_z \subseteq A \right\}$$

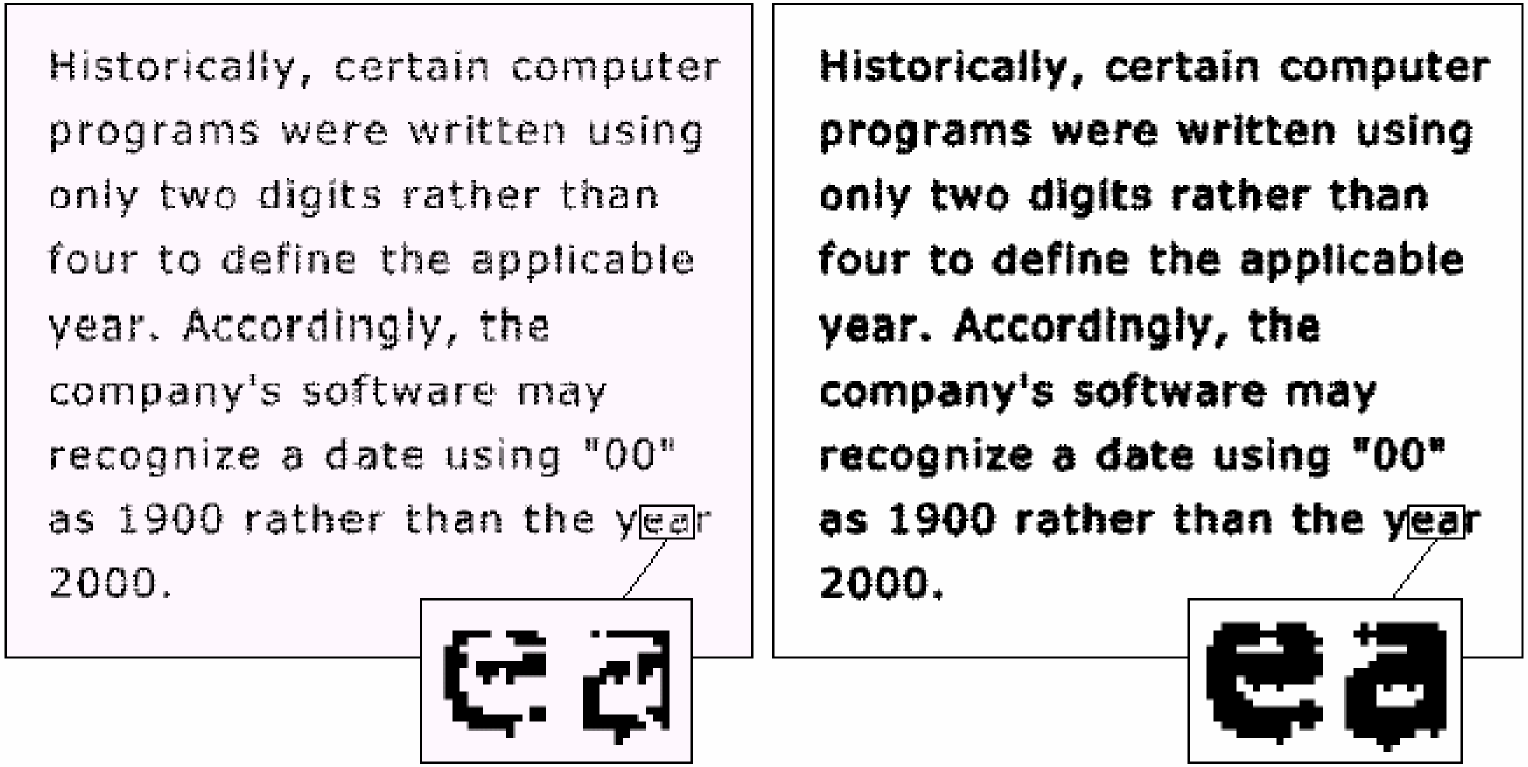
$$\left(A \Theta B\right)^c = A^c \oplus \hat{B}$$

$$\left(A \Theta B\right)^c = \left\{z \mid \left(B\right)_z \subseteq A\right\}^c$$

- If set $(B)_z$ is contained in set A, then $\left(B\right)_z \cap A^c = \Phi$,in which case the preceding equation becomes

$$\left(A \Theta B\right)^c = \left\{z \mid \left(B\right)_z \cap A^c = \Phi\right\}^c$$

$$\left(A \Theta B\right)^c = \left\{z \mid \left(B\right)_z \cap A^c = \Phi\right\} = A^c \oplus \hat{B}$$



0	1	0
1	1	1
0	1	0

a b c

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

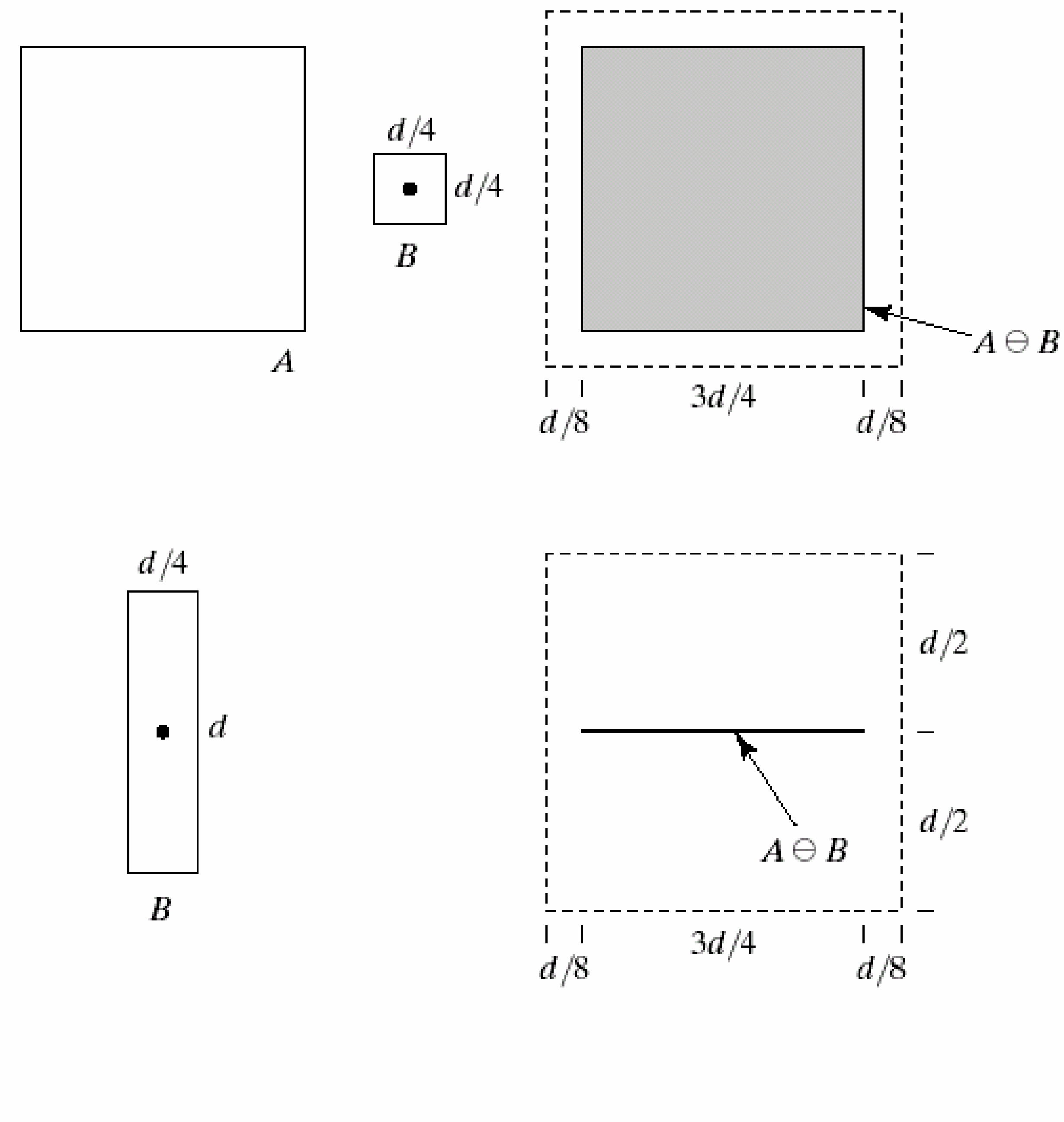
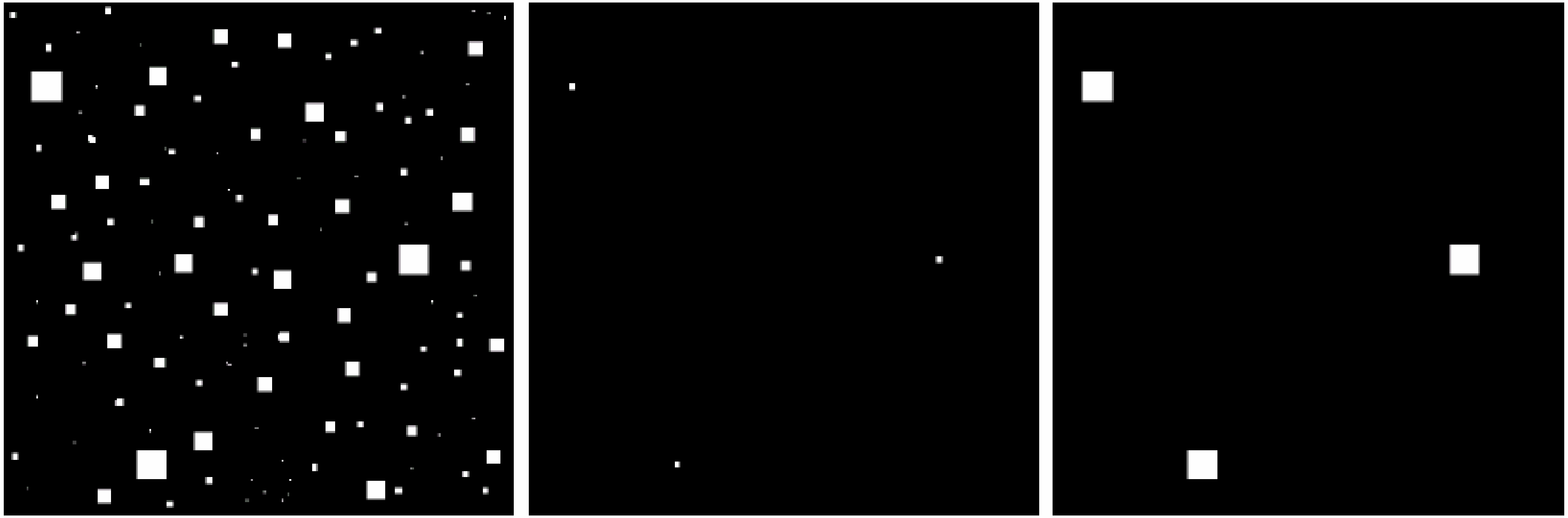
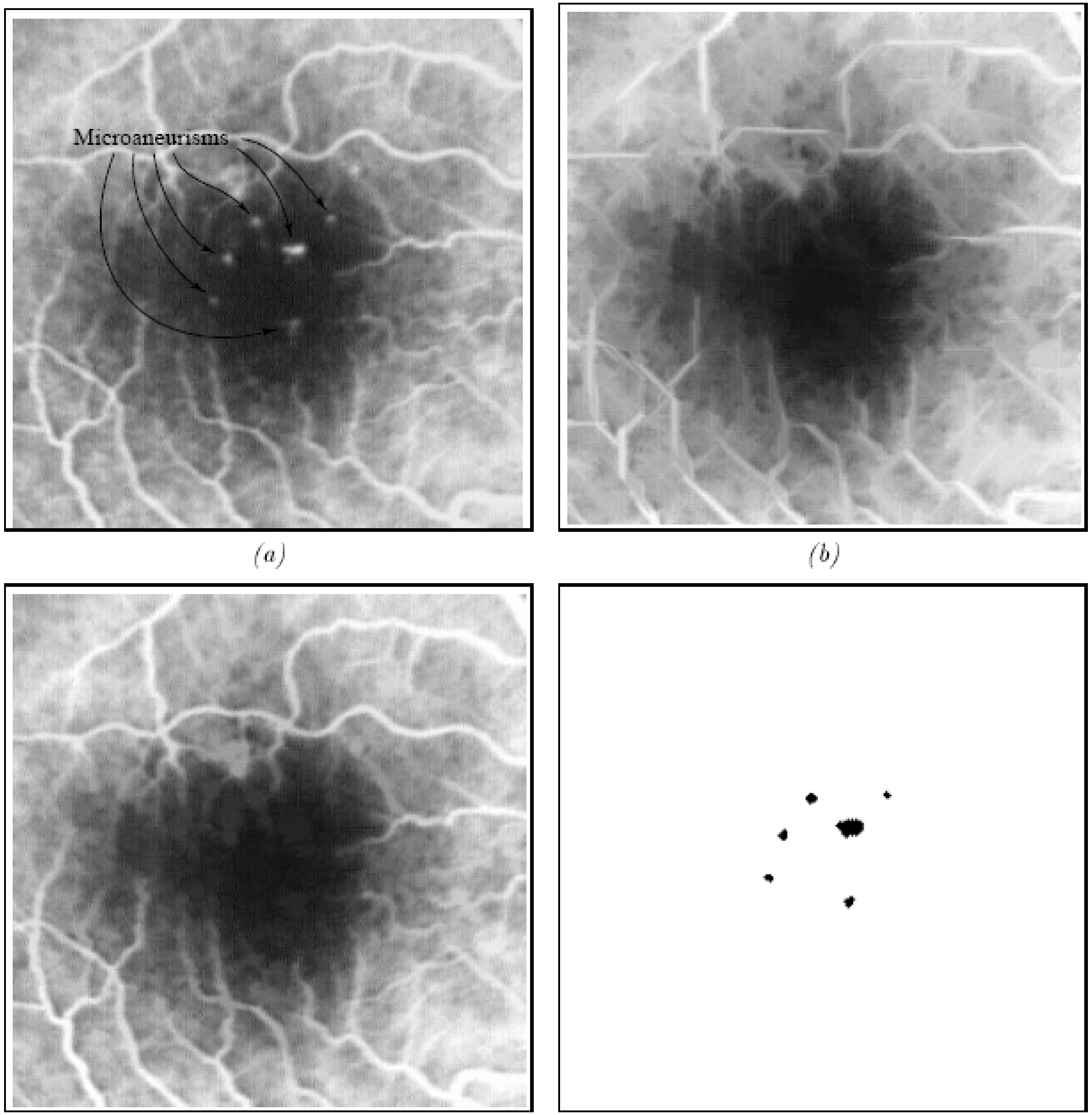


FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.



(a) (b) (c) (d)

Figure 10: *Use of grayscale reconstruction for image segmentation: (a) original image of blood vessels, (b) supremum of openings by segments, (c) reconstructed image, (d) microaneurisms obtained after substraction of (c) from (a) and thresholding step.*

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- Opening generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions.

- Closing also tends to smooth sections of contours but, eliminates small holes, and fills gaps in the contour.

- The opening of set A by structuring element B, denoted $A \circ B$, is defined as

$$A \circ B = (A \Theta B) \oplus B$$

- The opening of set A by structuring element B, denoted $A \bullet B$, is defined as

$$A \bullet B = (A \oplus B) \Theta B$$

- Opening can be expressed as a fitting process such that

$$A \circ B = \bigcup \{ (B)_z \mid (B)_z \subseteq A \}$$

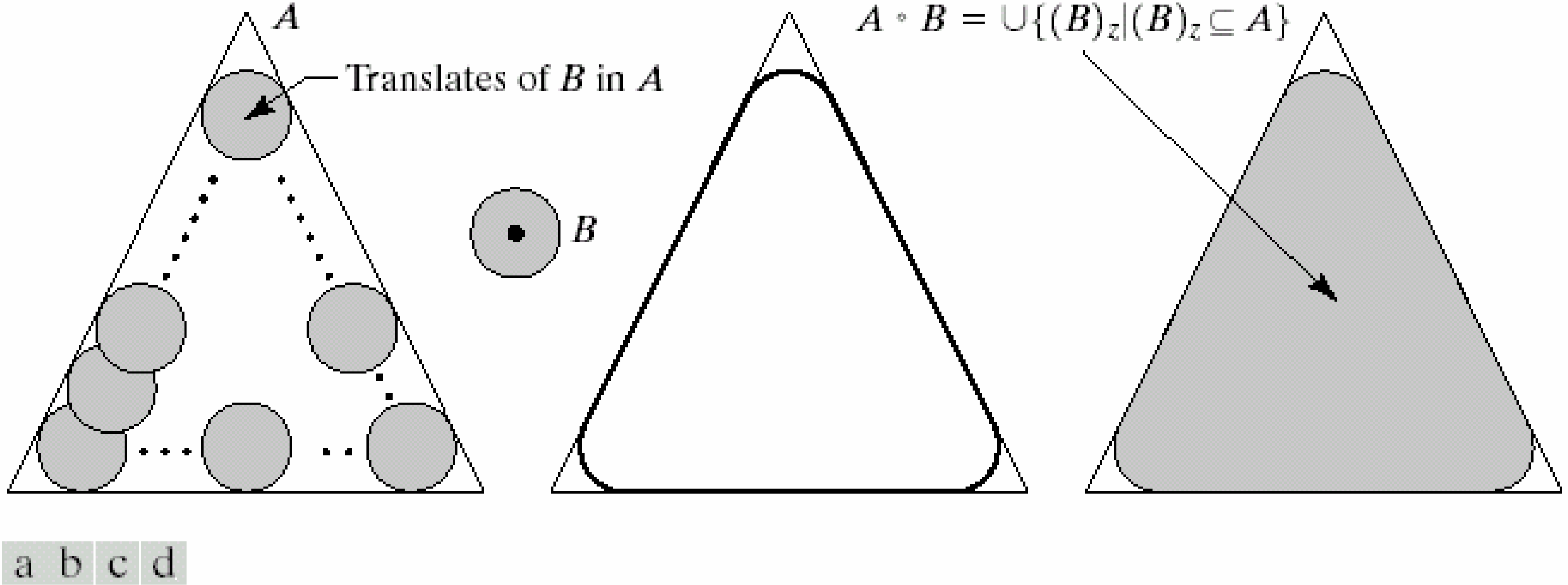
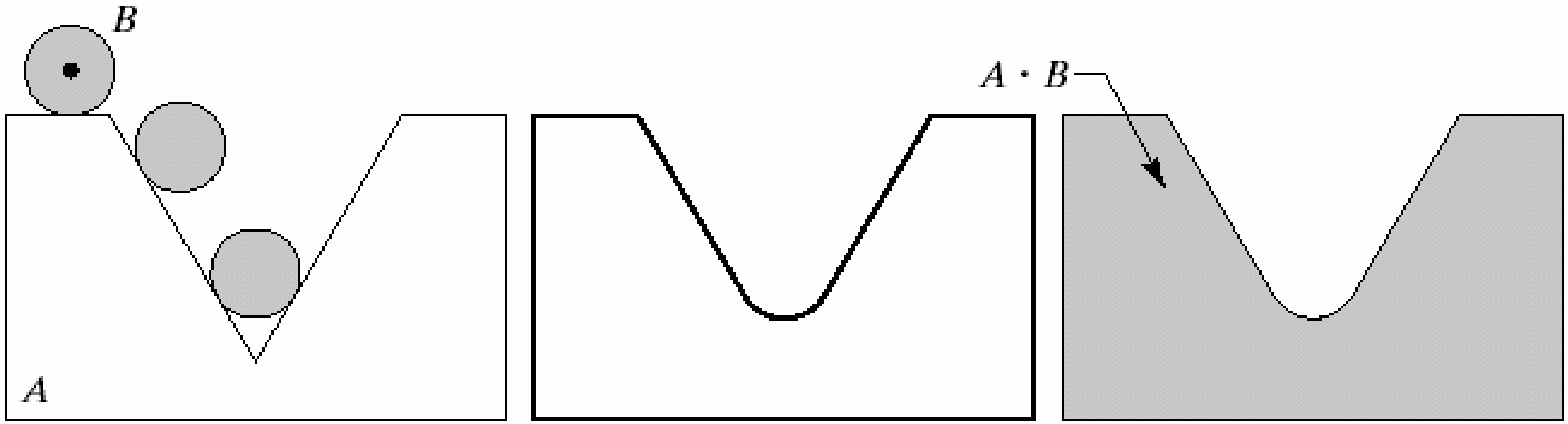


FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

- Opening and closing are duals of each other with respect to set complementation and reflection. That is,

$$(A \bullet B)^c = \left(A^c \circ \hat{B}\right)$$

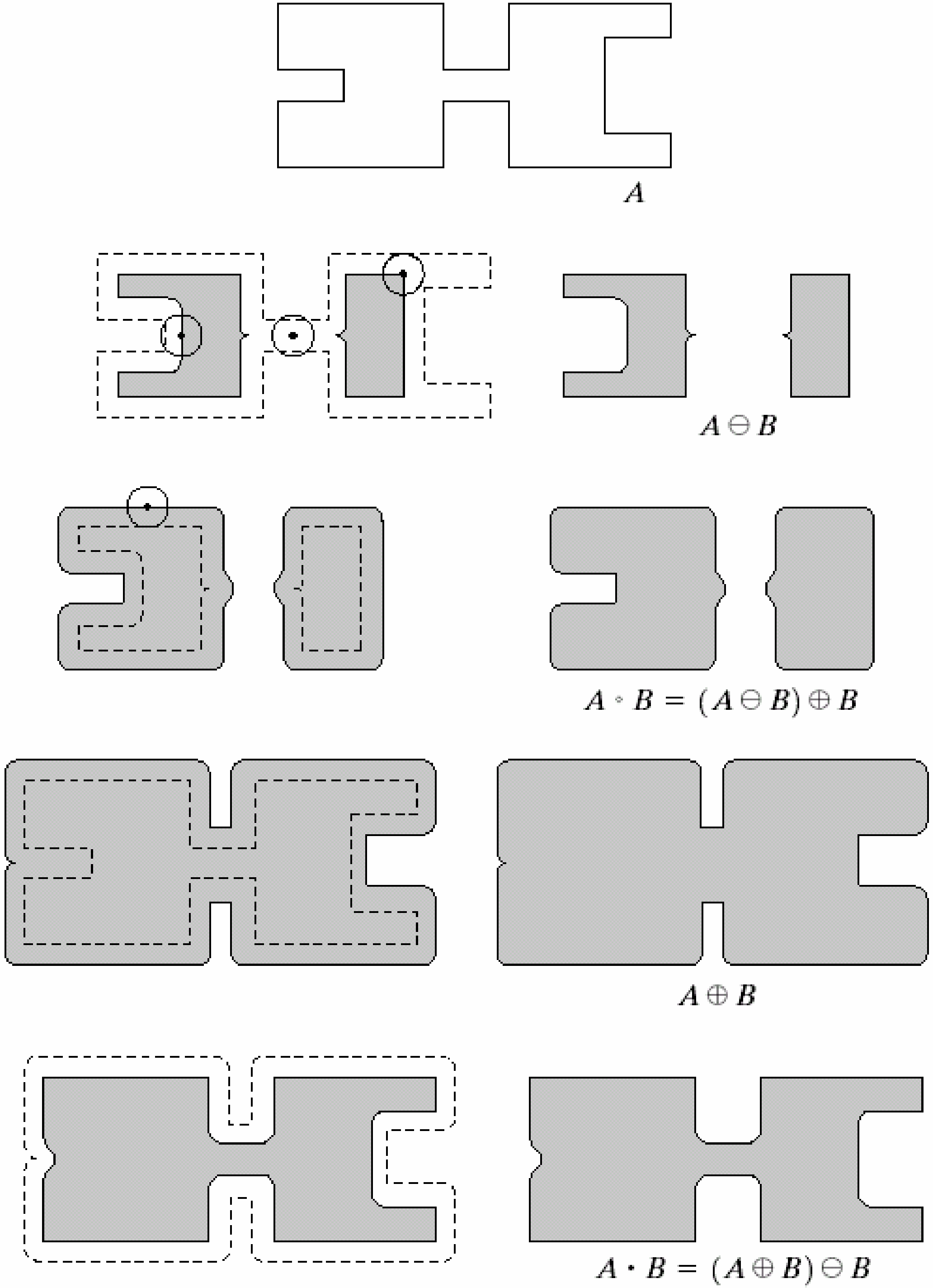


a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.



- The opening operation satisfies the following properties:
  - $A \circ B$ is a subset (subimage) of A.
  - If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.
  - $(A \circ B) \circ B = A \circ B$

- The closing operation satisfies the following properties:
  - A is a subset (subimage) of $A \bullet B$
  - If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$.
  - $(A \bullet B) \bullet B = A \bullet B$

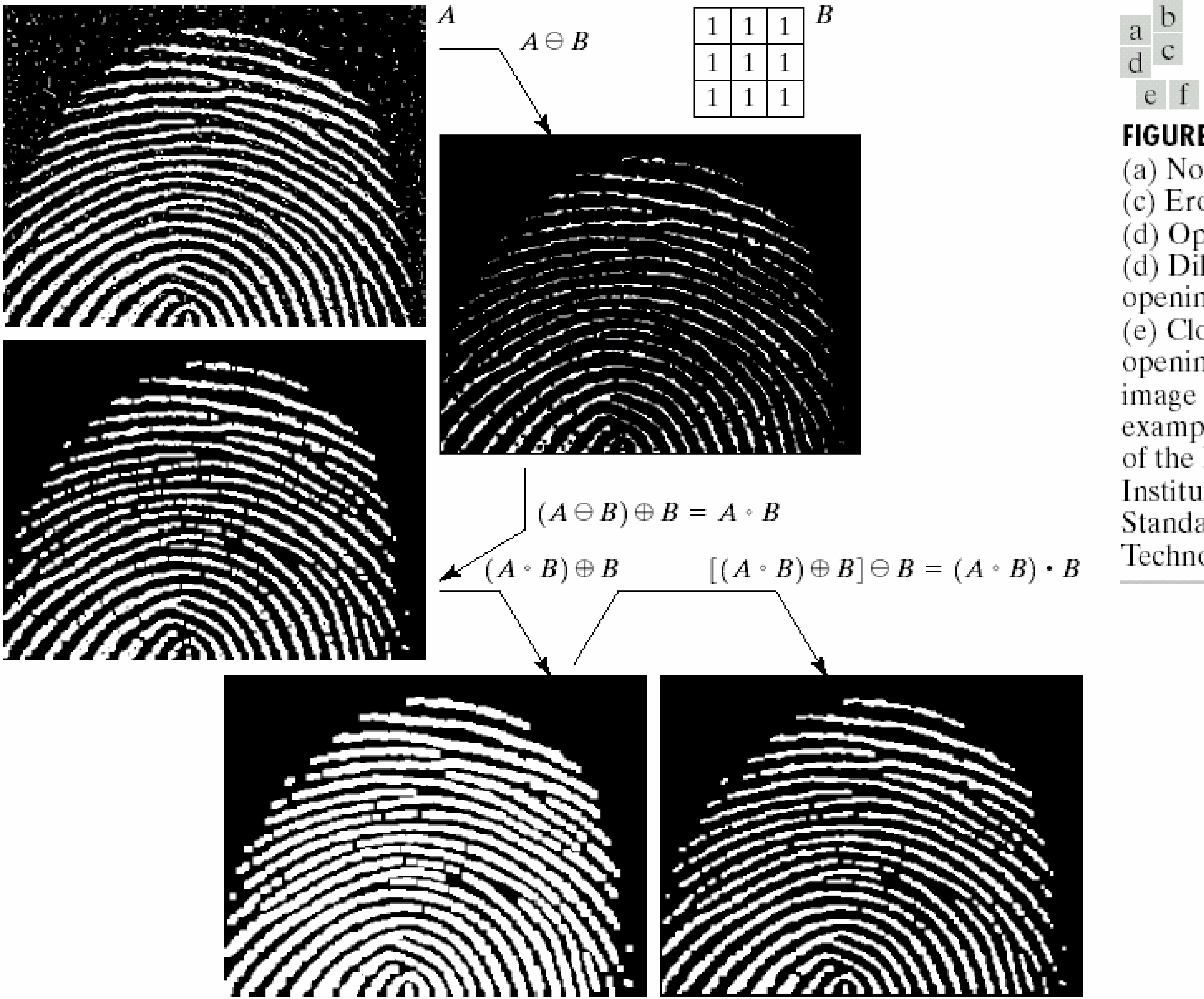


FIGURE 9.11
(a) Noisy image.
(c) Eroded image.
(d) Opening of A.
(d) Dilation of the opening.
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

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- $$A \circledast B = (A \Theta X) \cap \left[A^c \Theta (W - X) \right]$$
- If B_1=X and B_2=(W-X).

$$A \circledast B = \left(A \Theta B_1 \right) \cap \left(A^c \Theta B_2 \right)$$

- $$A \circledast B = \left(A \Theta B_1 \right) - \left(A \oplus \hat{B}_2 \right)$$

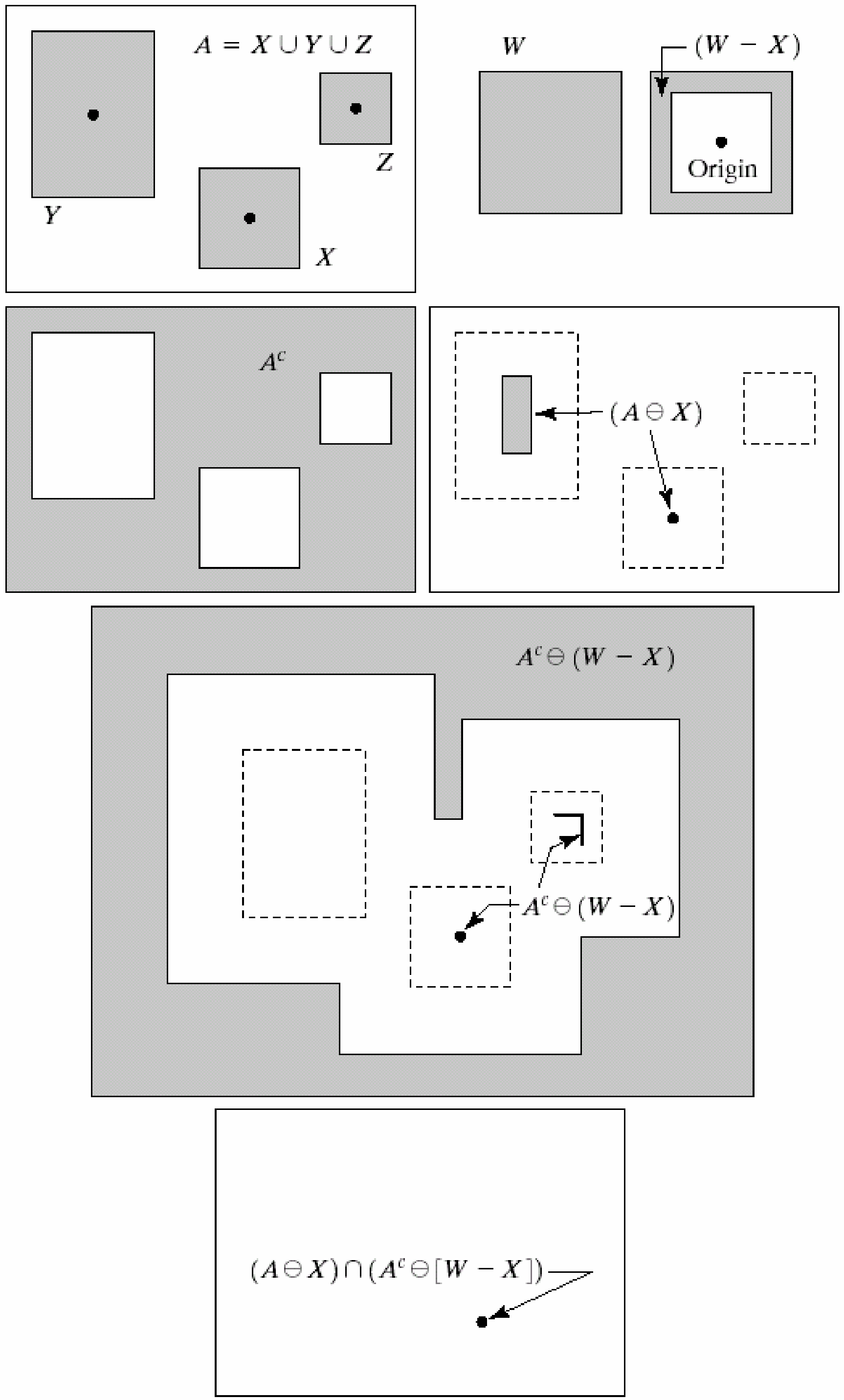

FIGURE 9.12
(a) Set A. (b) A window, W, and the local background of X with respect to W, $(W - X)$.
(c) Complement of A. (d) Erosion of A by X.
(e) Erosion of A^c by $(W - X)$.
(f) Intersection of (d) and (e), showing the location of the origin of X, as desired.

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- Boundary Extraction
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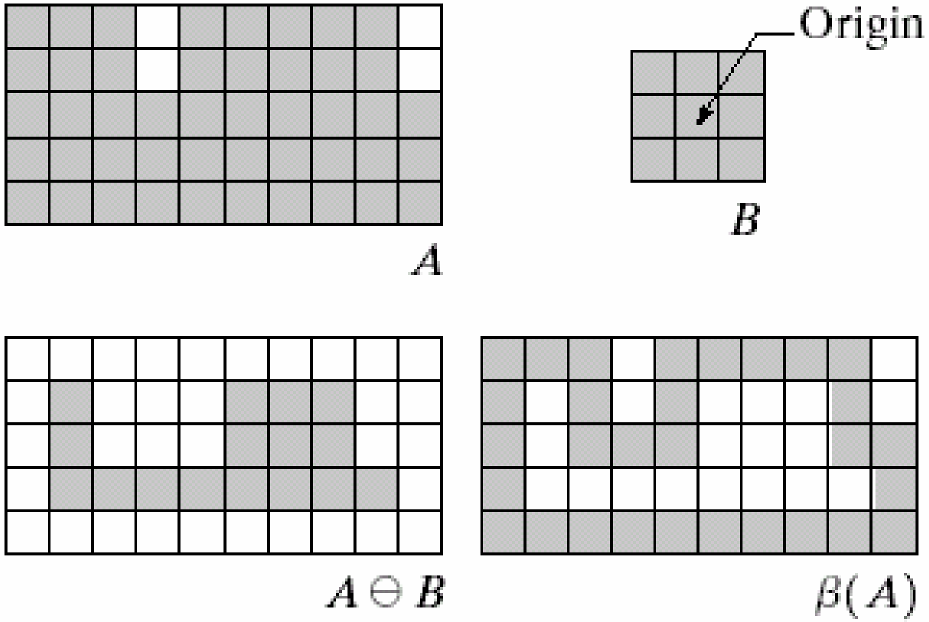
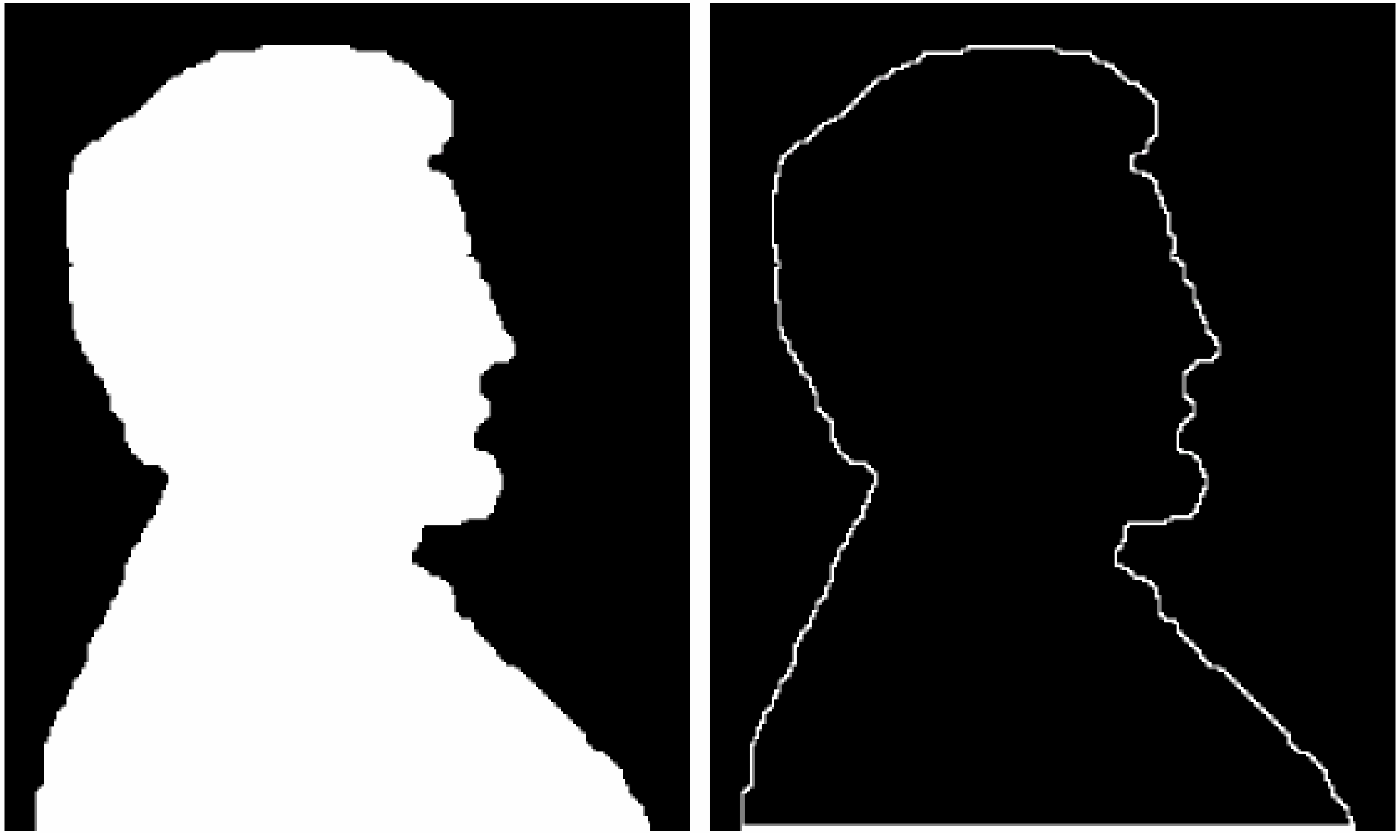
$$\beta(A) = A - (A \Theta B)$$

- Where B is a suitable structuring element.



a b
c d

FIGURE 9.13 (a) Set *A*. (b) Structuring element *B*. (c) *A* eroded by *B*. (d) Boundary, given by the set difference between *A* and its erosion.



a b

FIGURE 9.14
(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

- Boundary Extraction
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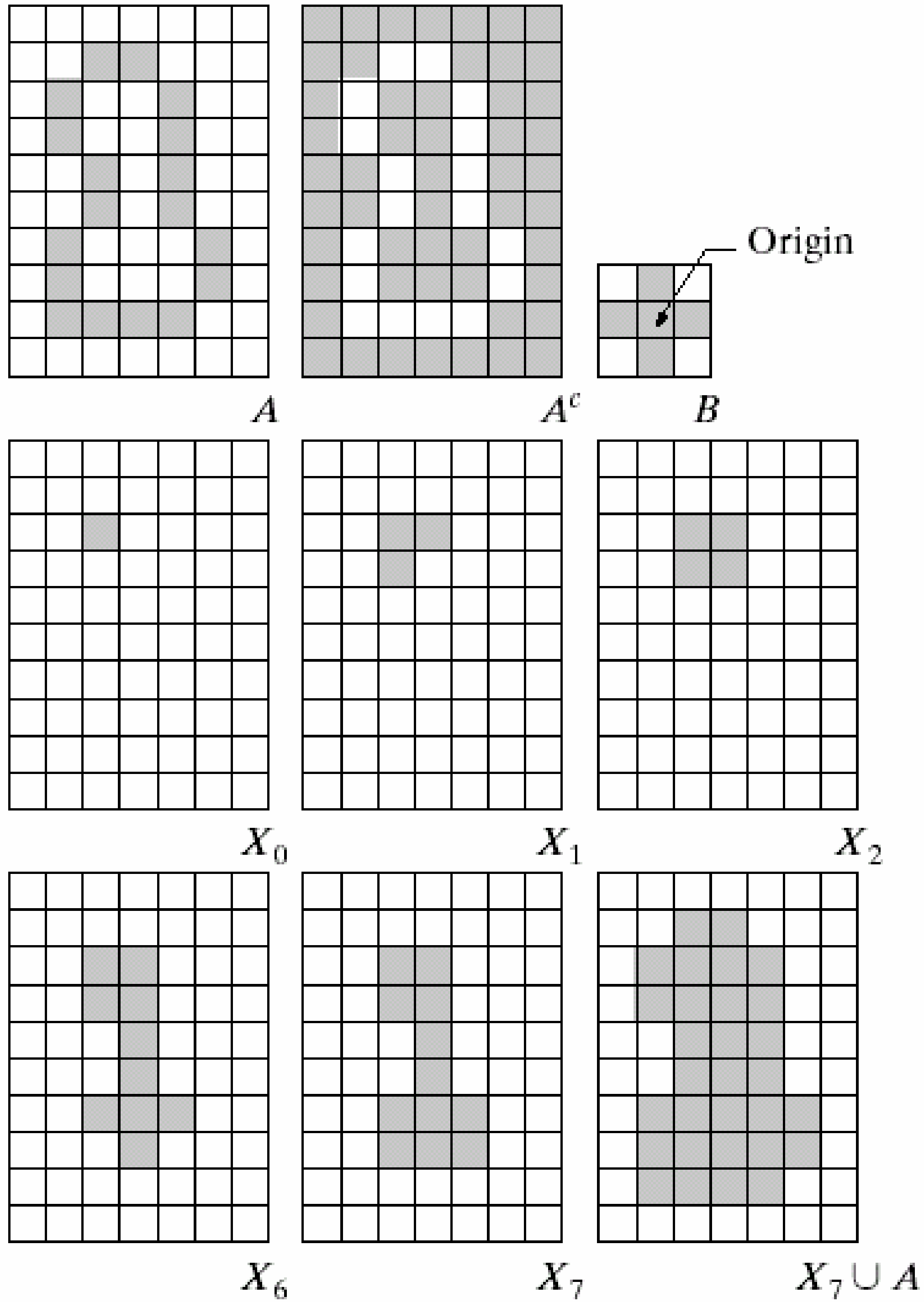
- The following procedure then fills the region with 1's:

$$X_k = \left(X_{k-1} \oplus B\right) \cap A^c, \qquad k = 1,2,3,....$$

- Where $X_0 = p$, and B is the symmetric structuring element.

a b c
d e f
g h i

FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].



- Boundary Extraction
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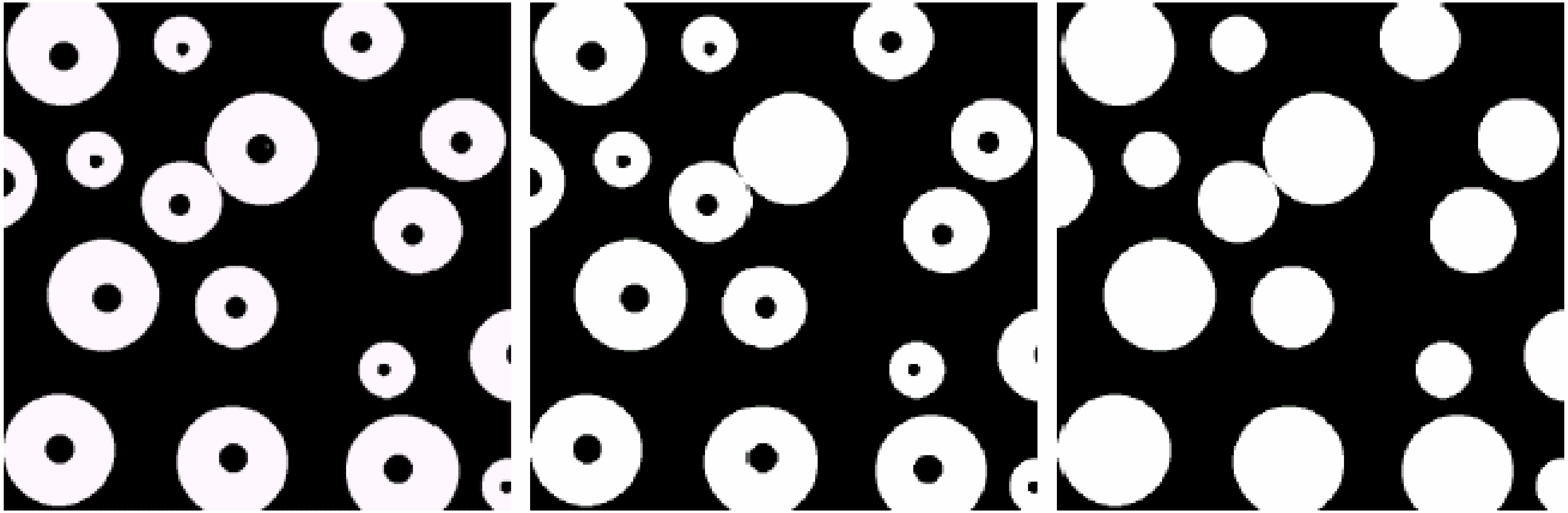
- Let Y represent a connected component contained in a set A and assume that a point p of Y is known. The following procedure then fills the region with 1's:

$$X_k = \left(X_{k-1} \oplus B\right) \cap A, \qquad k = 1,2,3,\ldots.$$

- Where $X_0 = p$, and B is the symmetric structuring element.



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

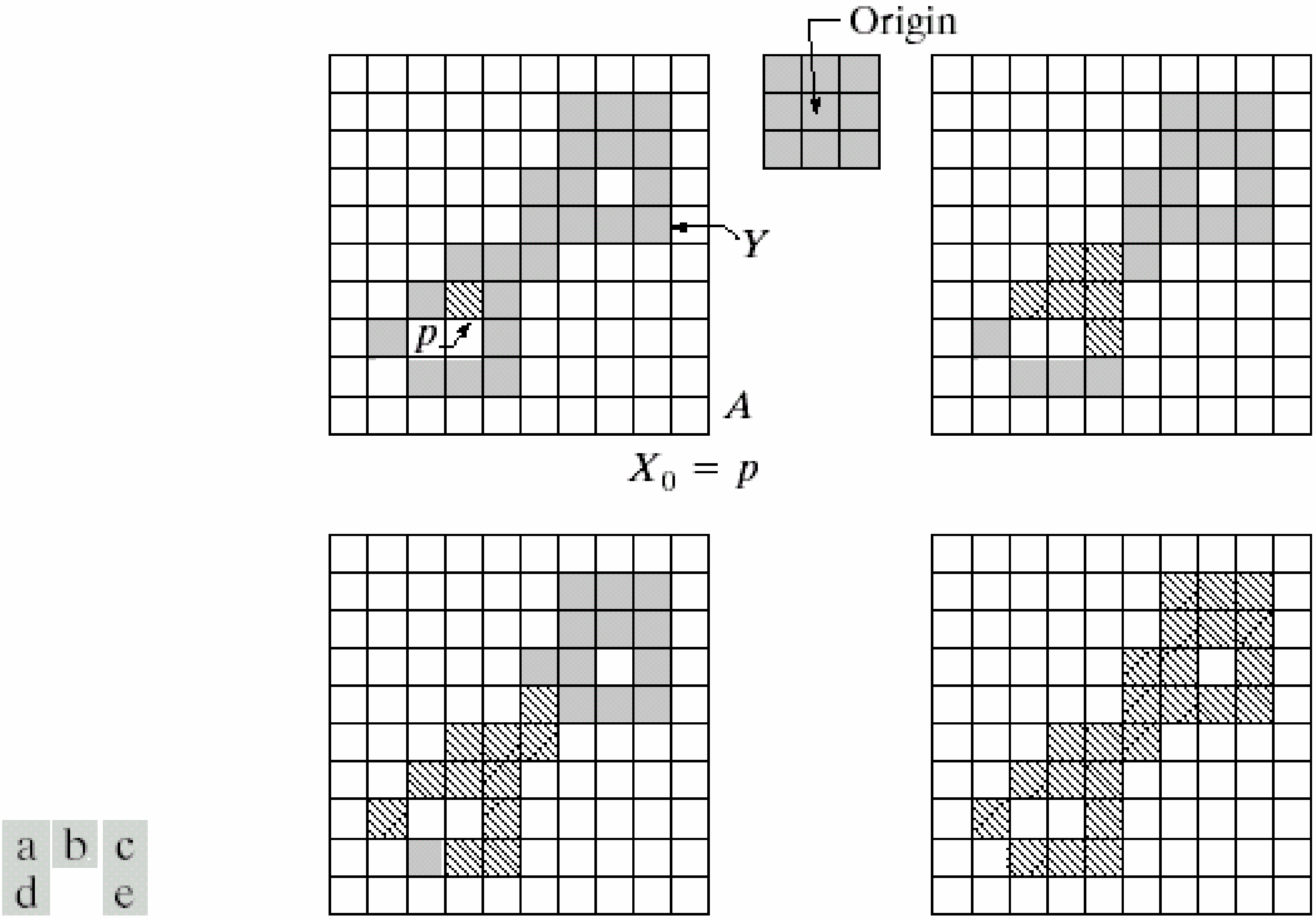


FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5 × 5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

- Boundary Extraction
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- The thinning of a set A by a structuring element B, denoted $A \otimes B$, can be defined in term of the hit-or0miss transform:

$$
\begin{aligned}
A \otimes B &= A - (A \circledast B) \\
&= A \cap (A \circledast B)^c .
\end{aligned}
$$

- A more useful expression for thinning A symmetrically is based on a sequence of structuring elements:

$$\{B\}=\{B^1,B^2,B^3,\ldots,B^n\}$$

- Where B^i is a rotated version of B^{i-1}. Using this concept, we now define thinning by a sequence of structuring elements as

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n).$$

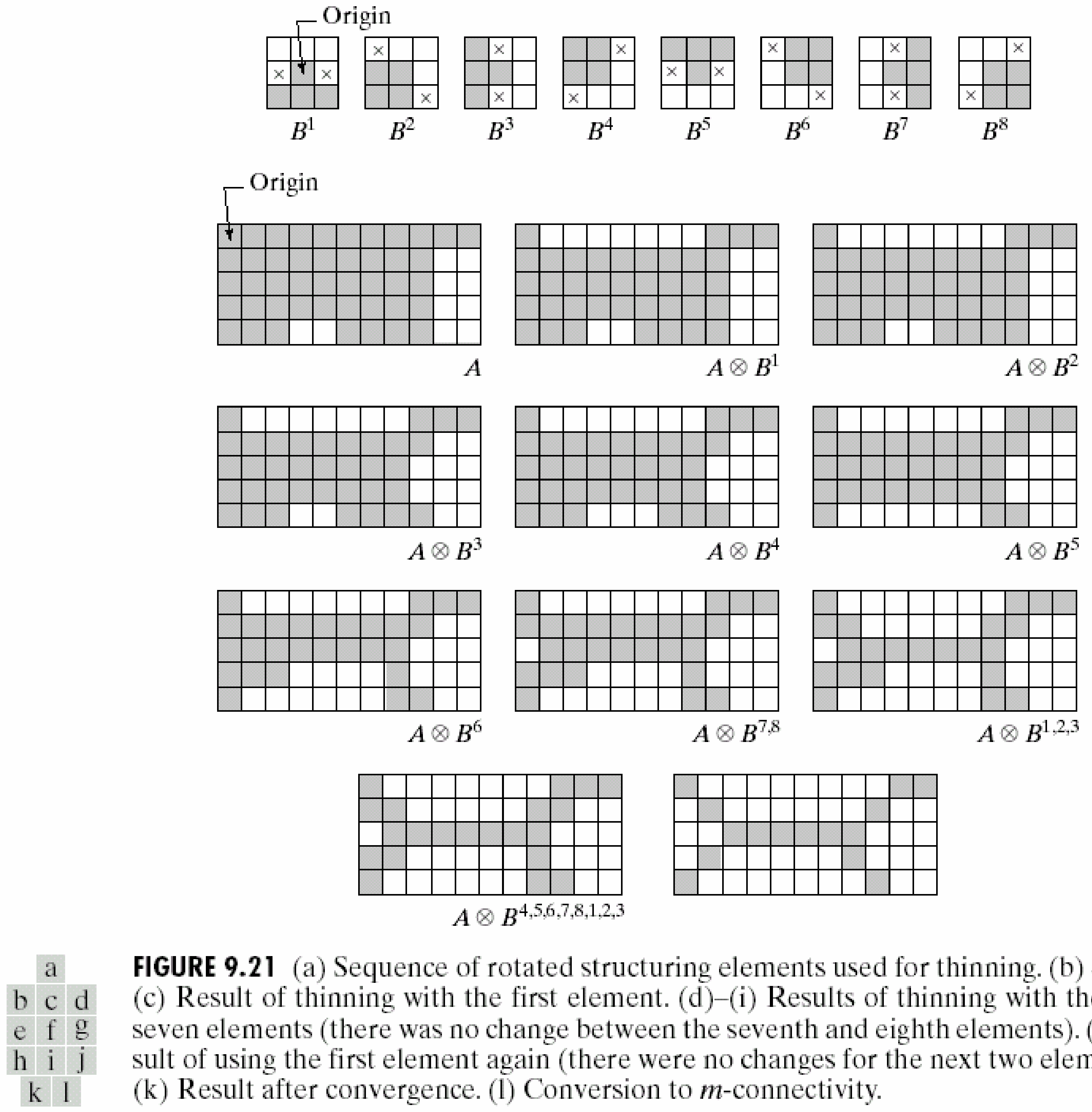

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to m-connectivity.

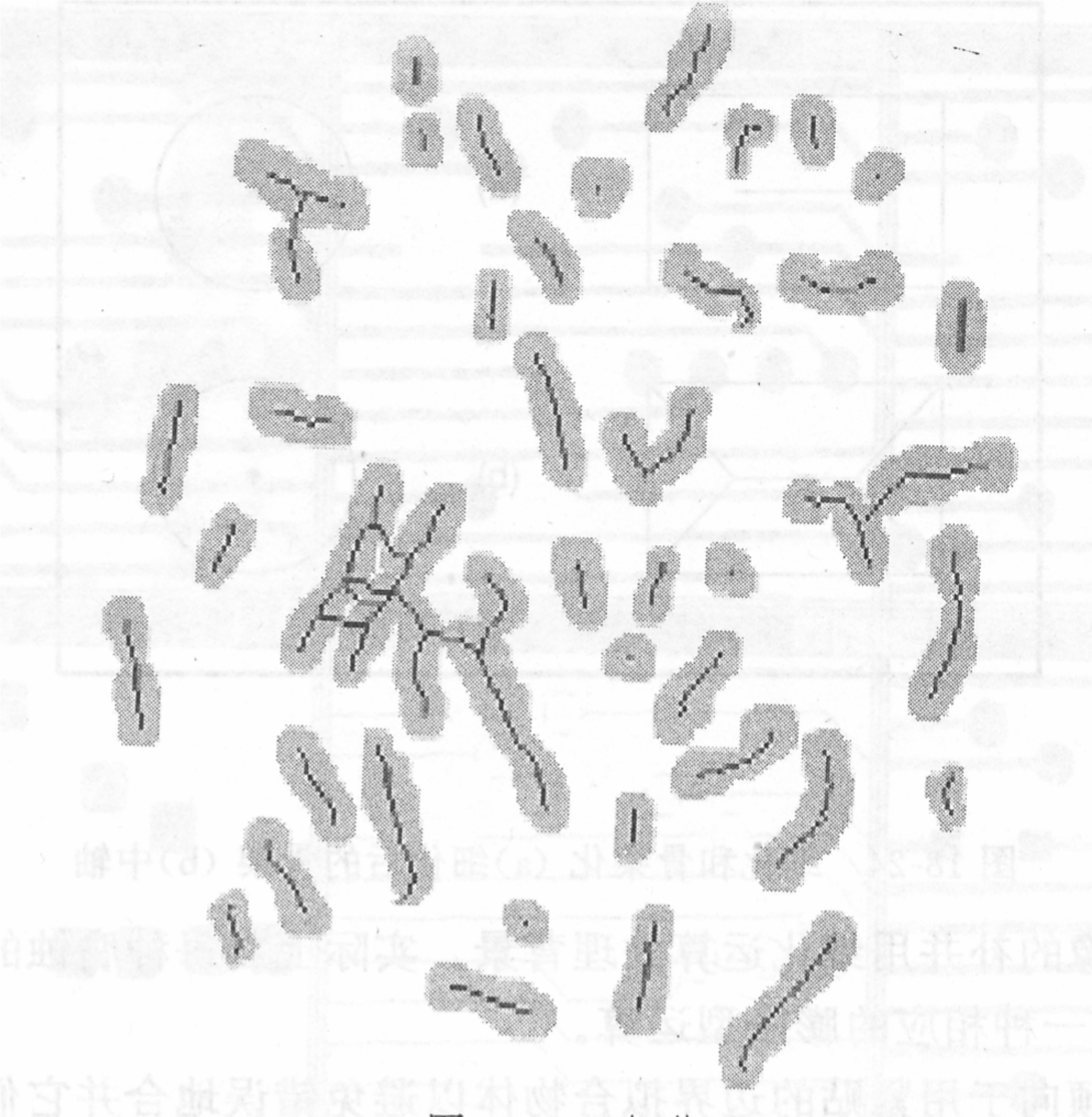

图 18-23　细化

- Boundary Extraction
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- Thickening is the morphological dual of thinning and is defined by the expression

$$A \odot B = A \cup (A \circledast B)$$

- Where B is a structuring element suitable for thickening. As in thinning, thickening can be defined as a sequential operation:

$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2) \ldots) \odot B^n).$$

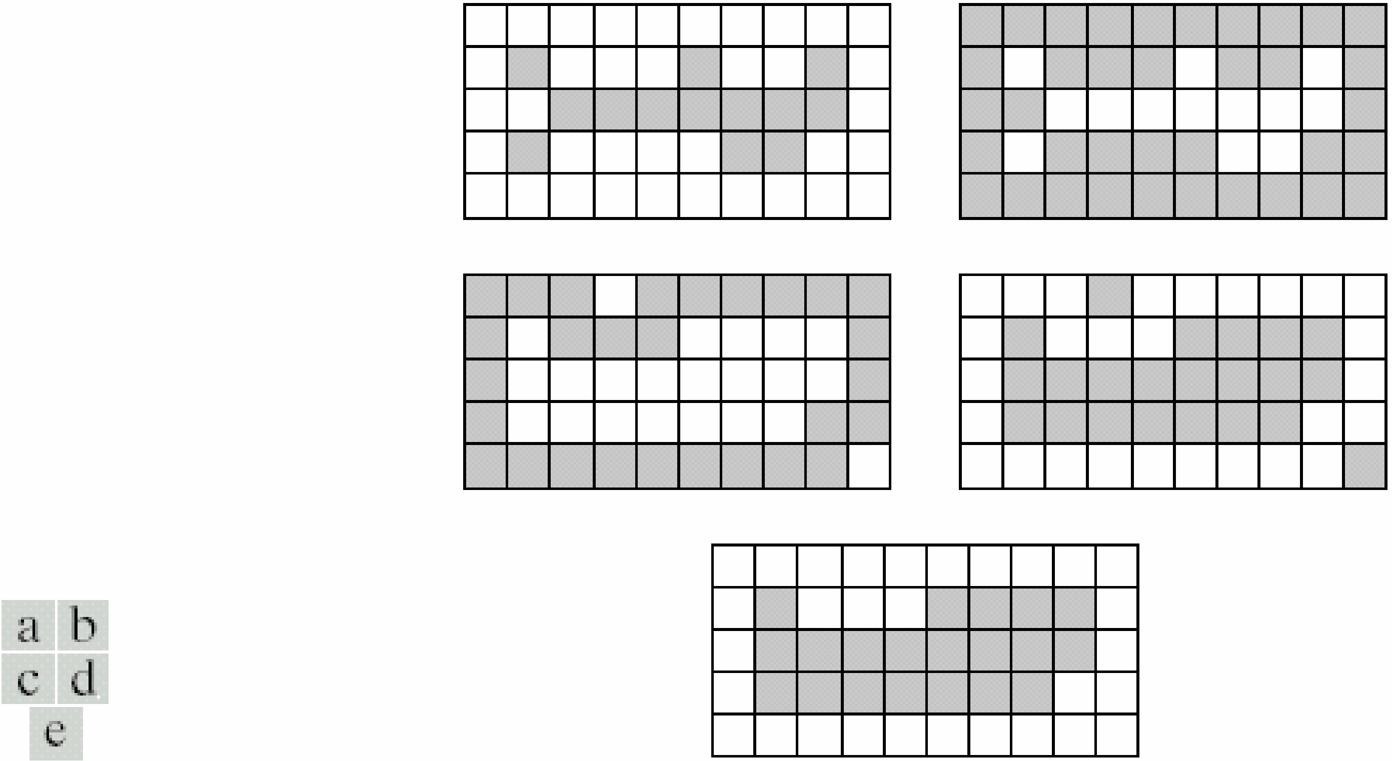

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

- Boundary Extraction
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- The skeleton of A can be expressed in terms of erosions and openings. That is, it can be shown that

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

with

$$S_k(A) = (A \Theta kB) - (A \Theta kB) \circ B$$

where B is a structuring element, and indicates k successive erosions of A:

$$\left(A \Theta kB\right) = \left(\ldots\left(\left(A \Theta B\right) \Theta B\right) \Theta \ldots\right) \Theta B$$

k times, and K is the last iterative step before A erodes to an empty set. In other words,

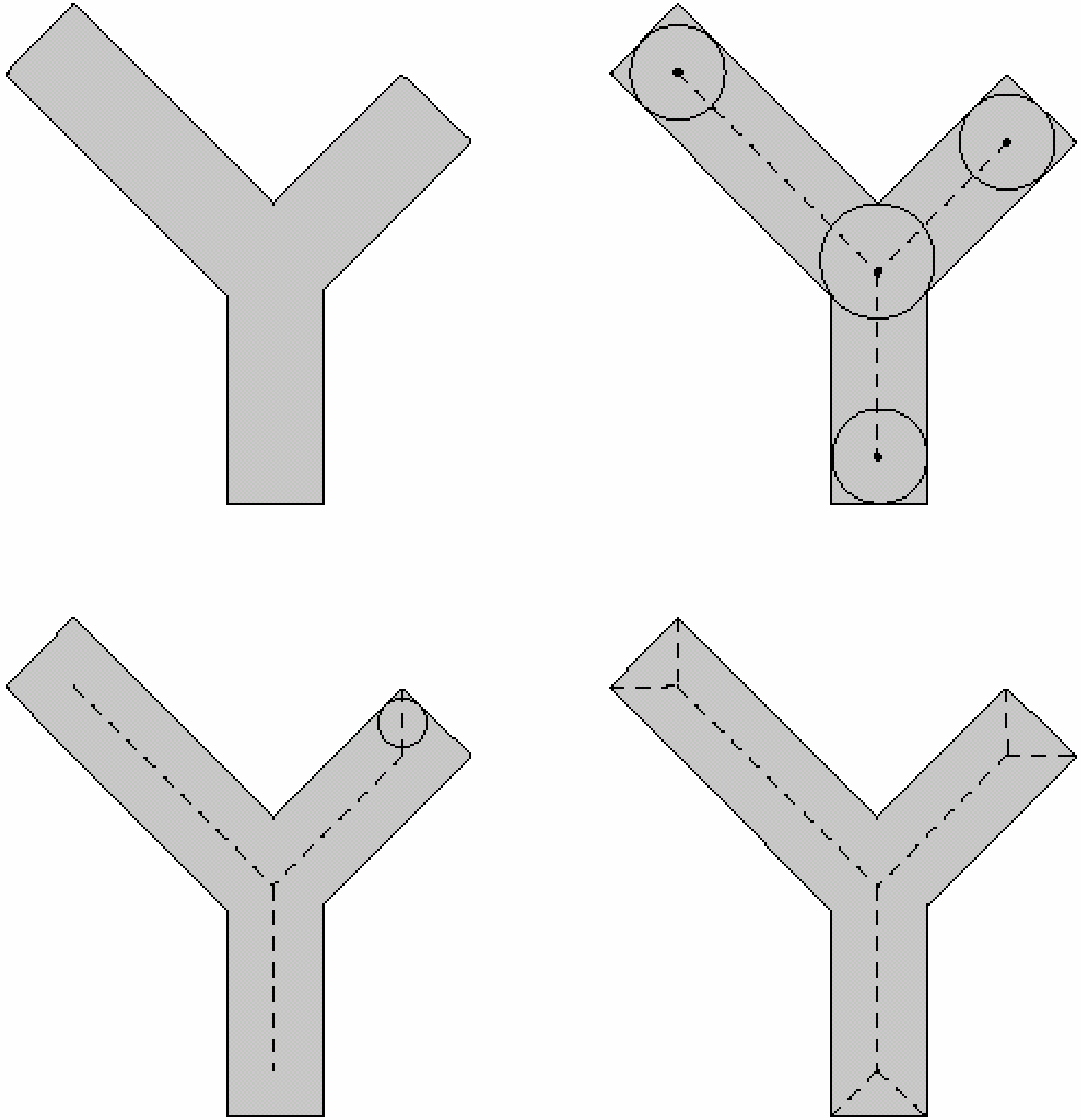
$$K = \max\left\{k \mid \left(A \Theta kB\right) \neq \Phi\right\}$$

- The formulation given above states that S(A) can be obtained as the union of the skeleton subsets $S_k(A)$. Also, it can be shown that A can be reconstructed from these subsets by using the equation:

$$A = \bigcup_{k=0}^{K} \left(S_k(A) \oplus kB\right)$$

where $\left(S_k(A) \oplus kB\right)$ denotes k successive dilations of $S_k(A)$; that is

$$\left(S_k(A) \oplus kB\right) = \left(\left(\ldots\left(S_k\left(A\right) \oplus B\right) \oplus B\right) \oplus \ldots\right) \oplus B$$



a b
c d

FIGURE 9.23
(a) Set A.
(b) Various positions of maximum disks with centers on the skeleton of A.
(c) Another maximum disk on a different segment of the skeleton of A.
(d) Complete skeleton.

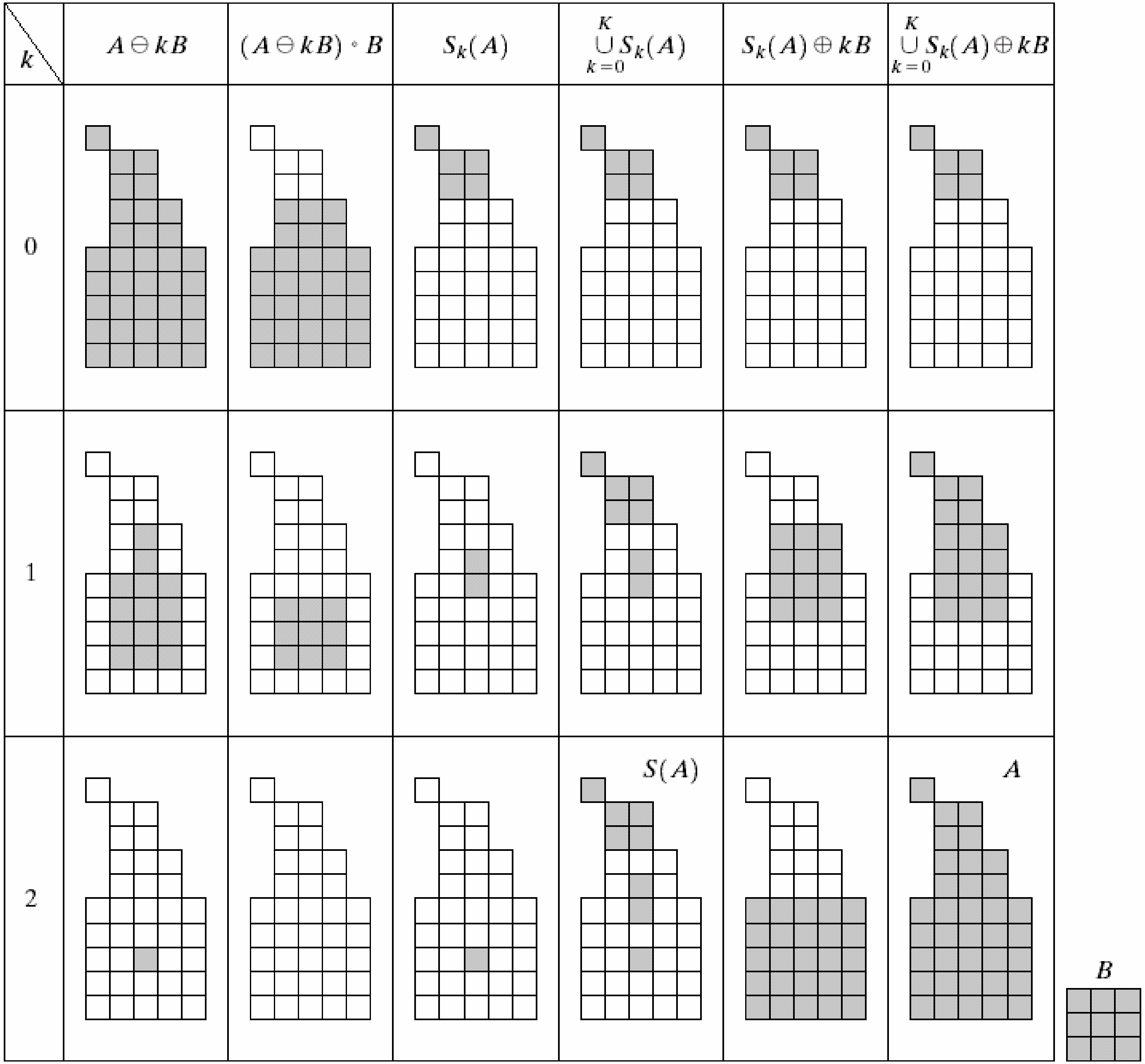


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

- Boundary Extraction
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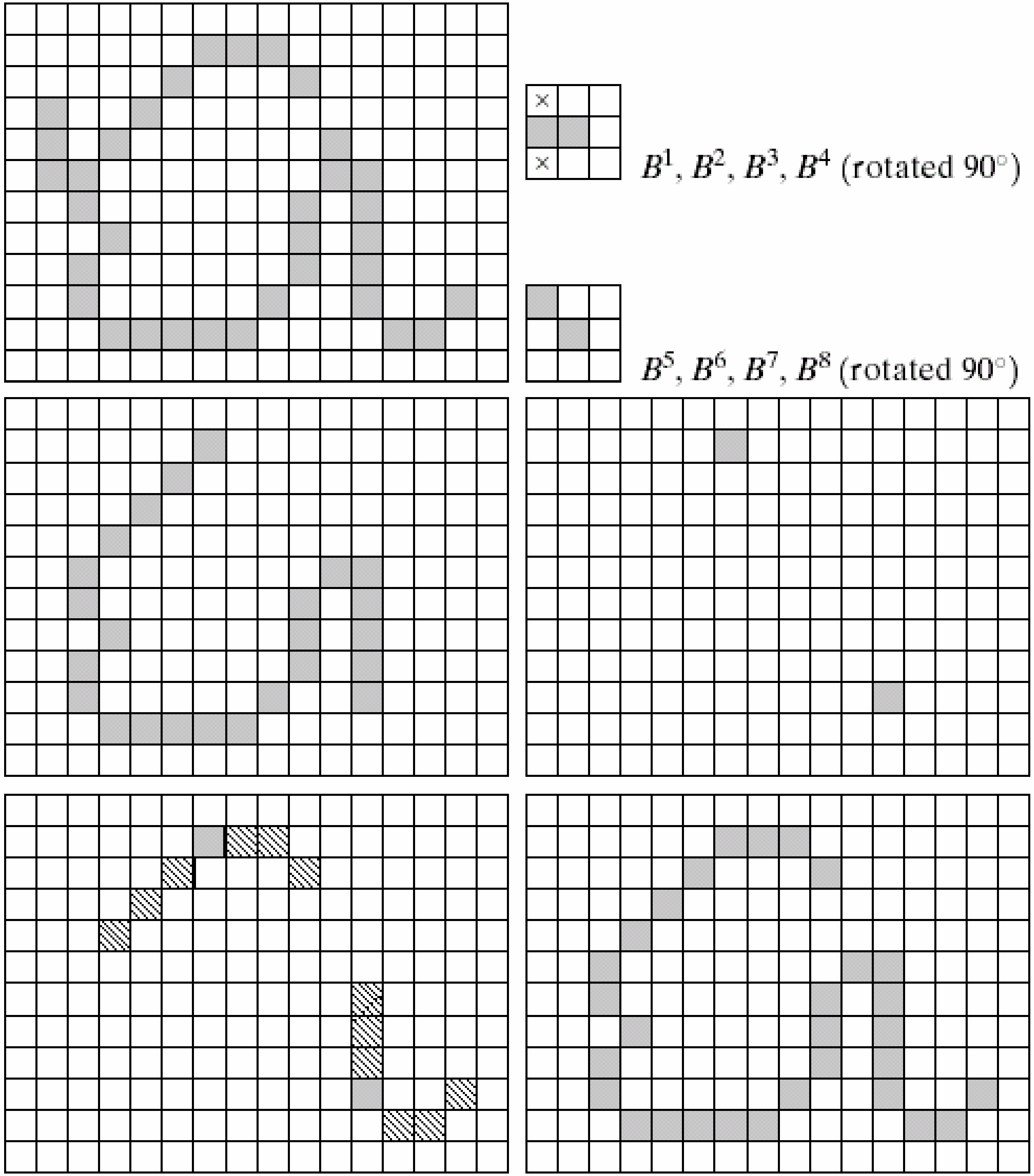

FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

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TABLE 9.2
Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c; X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A; X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A; i = 1, 2, 3, 4;$ $k = 1, 2, 3, \ldots; X_0^i = A$; and $D^i = X_{\text{conv}}^i$.	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

TABLE 9.2
Summary of morphological results and their properties. *(continued)*

TABLE 9.2 Summary of morphological results and their properties. *(continued)*

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.

Skeletons	$S(A) = \bigcup_{k=0} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB)$ $- [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.

TABLE 9.2
Summary of morphological results and their properties. *(continued)*

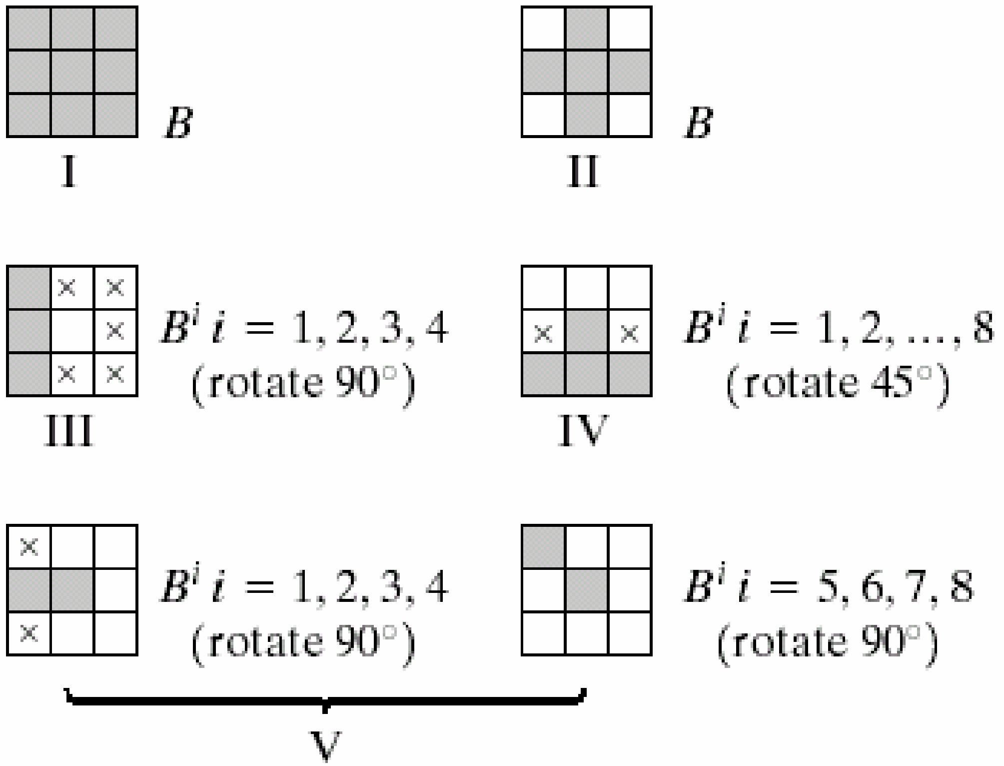
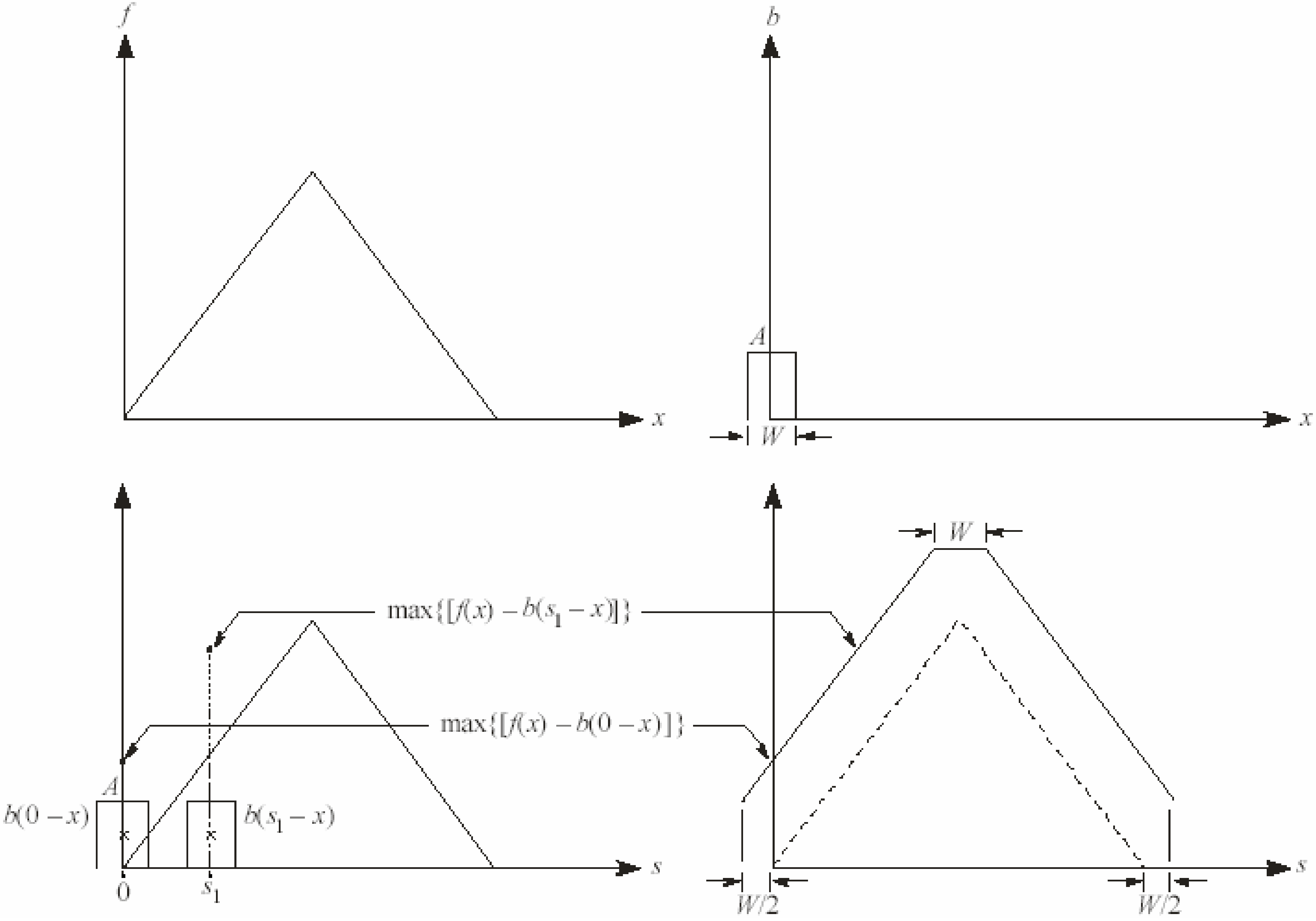


FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

- Preliminaries
- Dilation and Erosion
- Opening and Closing
- The Hit-or-Miss Transformation
- Some Basic Morphological Algorithms
- Extensions to Gray-Scale Images

- Dilation
- Erosion
- Opening and Closing
- Some Applications of Gray-Scale Morphology


f
x
b
A
W
x
max{[f(x) − b(s1 − x)]}
max{[f(x) − b(0 − x)]}
A
b(0 − x)
b(s1 − x)
0
s1
s
W
W/2
W/2
s

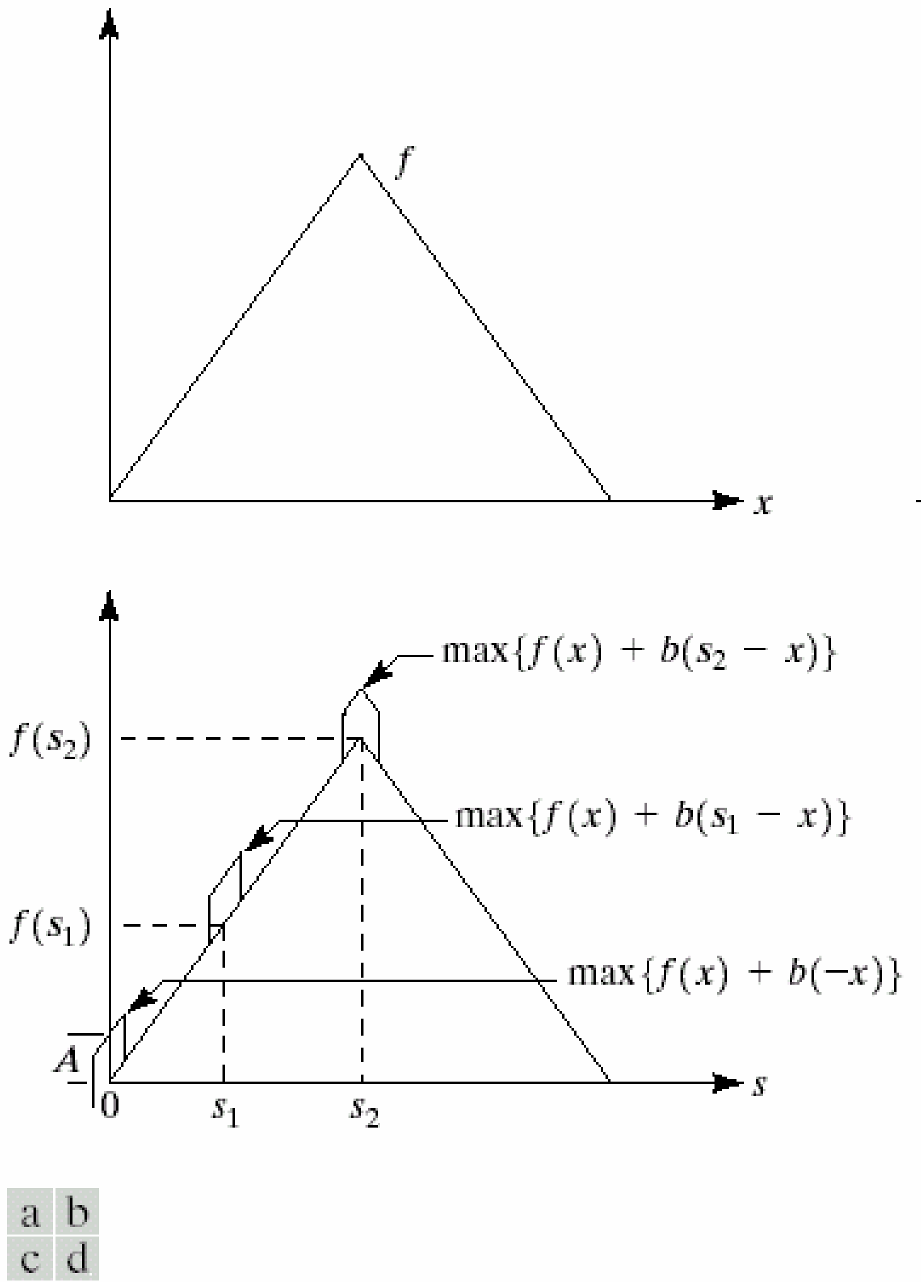


FIGURE 9.27 (a) A simple function. (b) Structuring element of height A. (c) Result of dilation for various positions of sliding b past f. (d) Complete result of dilation (shown solid).

- Dilation
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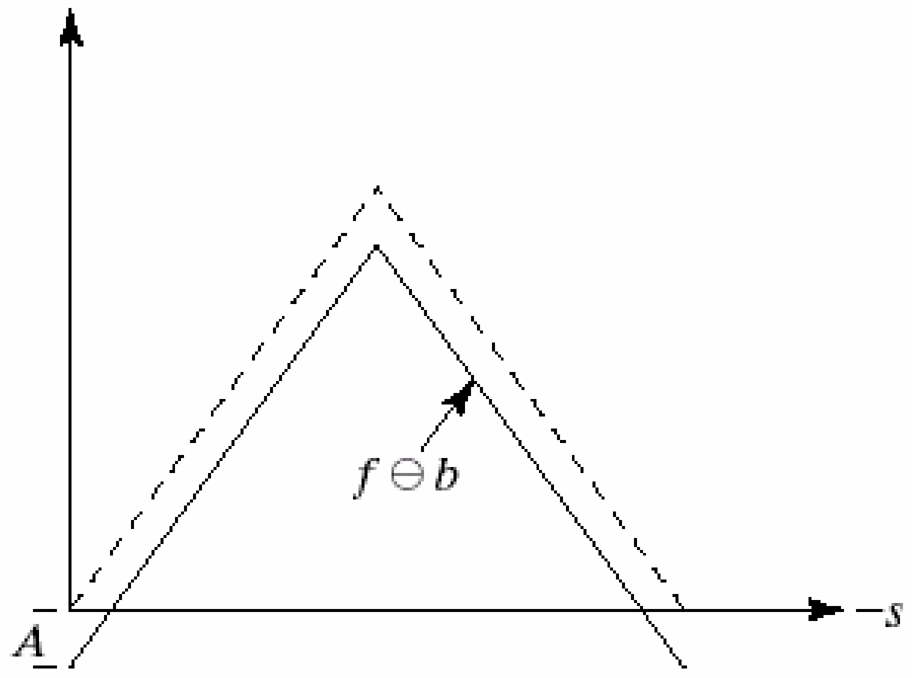
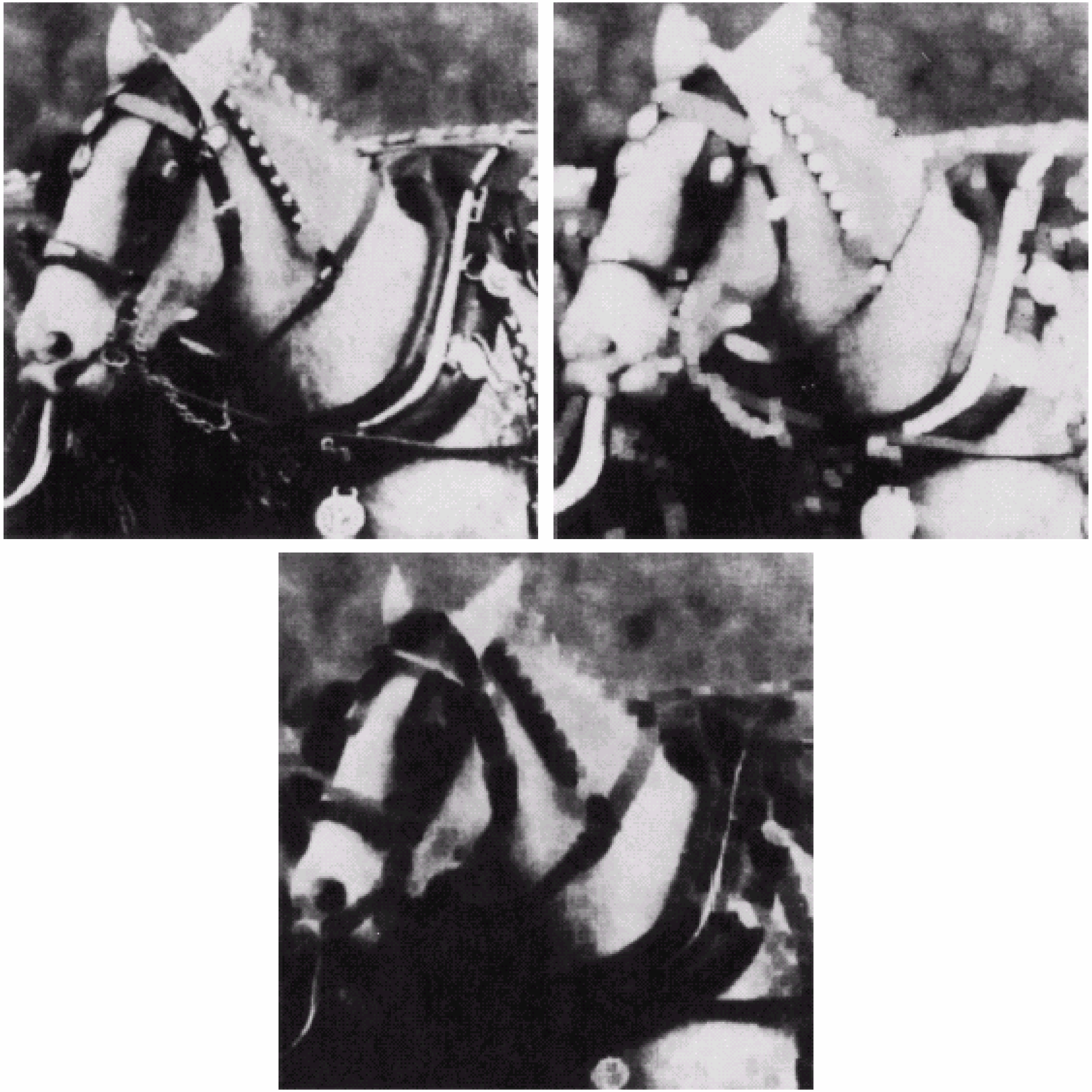
FIGURE 9.28
Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).





a b
c

FIGURE 9.29 (a) Original image. (b) Result of dilation. (c) Result of erosion. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

- Dilation
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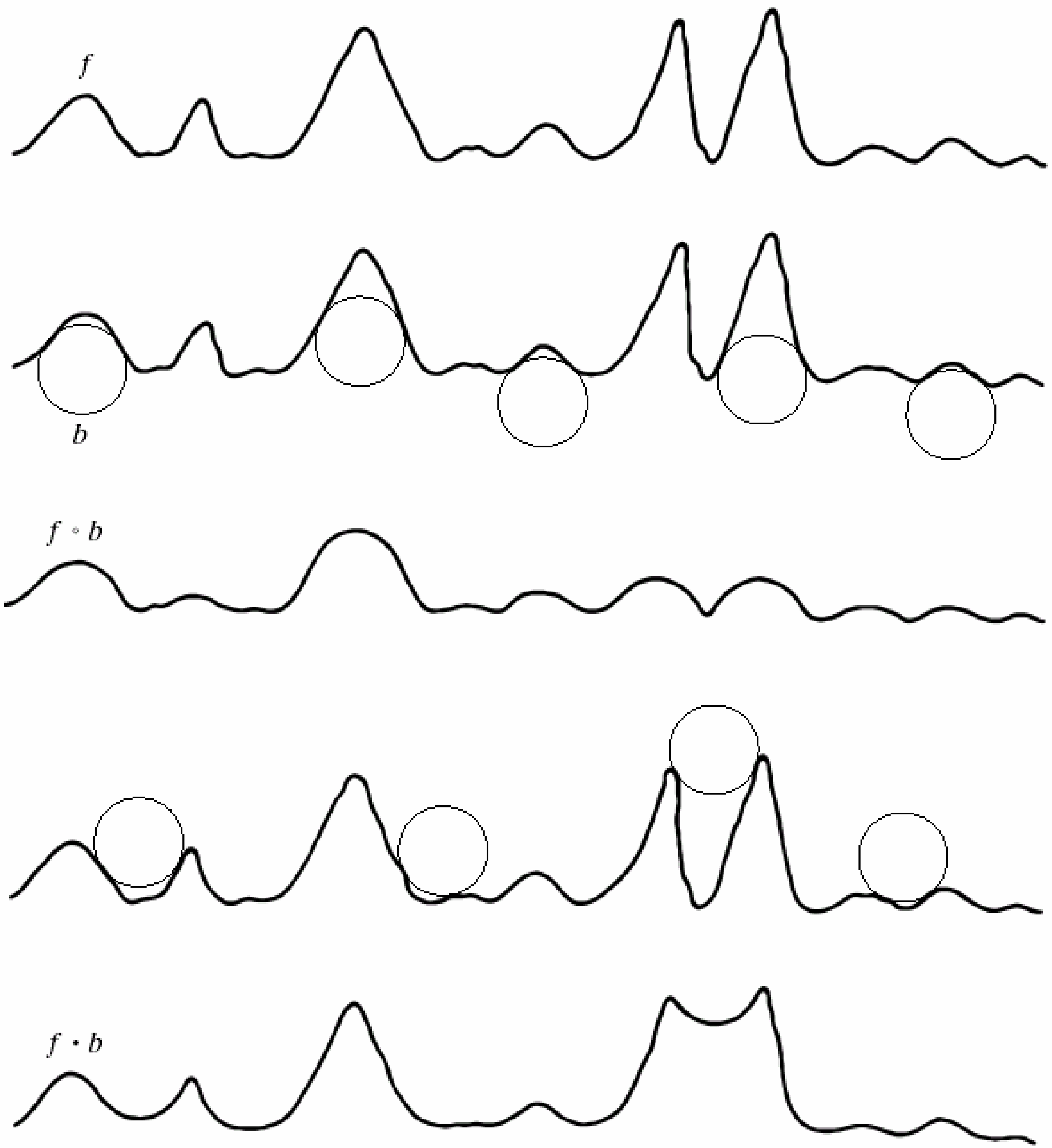


a
b
c
d
e

FIGURE 9.30
(a) A gray-scale scan line.
(b) Positions of rolling ball for opening.
(c) Result of opening.
(d) Positions of rolling ball for closing. (e) Result of closing.

a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

- Dilation
- Erosion
- Opening and Closing
- Some Applications of Gray-Scale Morphology

FIGURE 9.32 Morphological smoothing of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

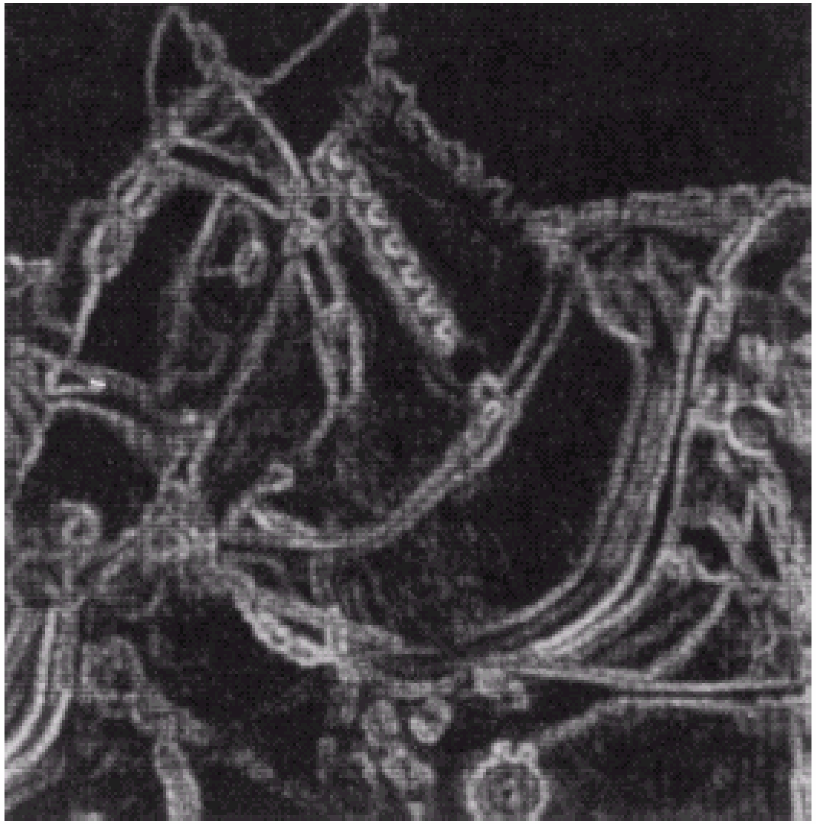


FIGURE 9.33 Morphological gradient of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

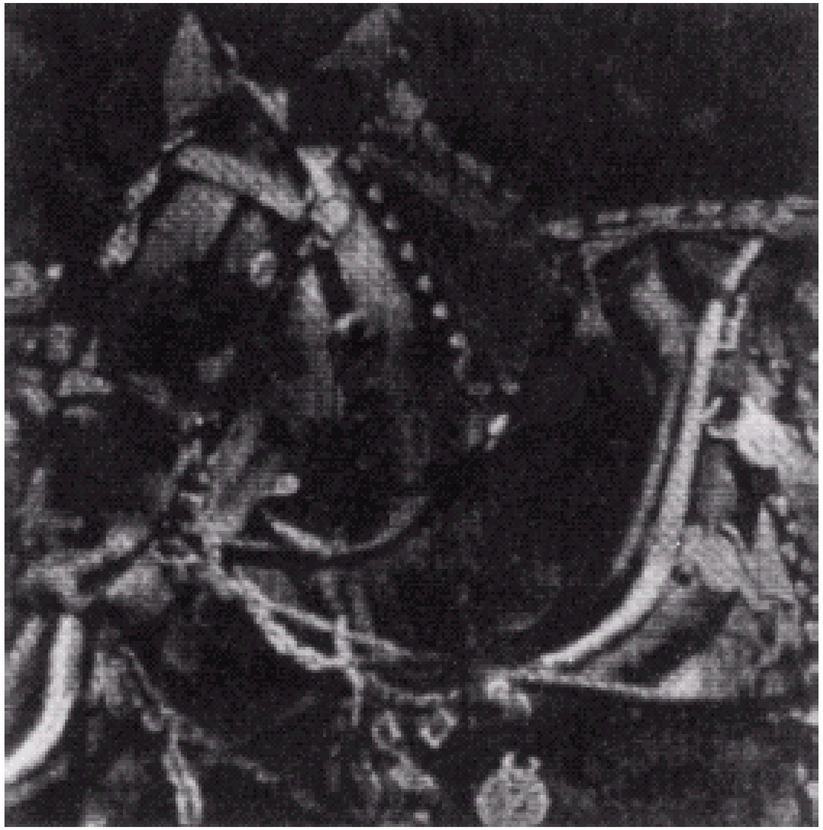
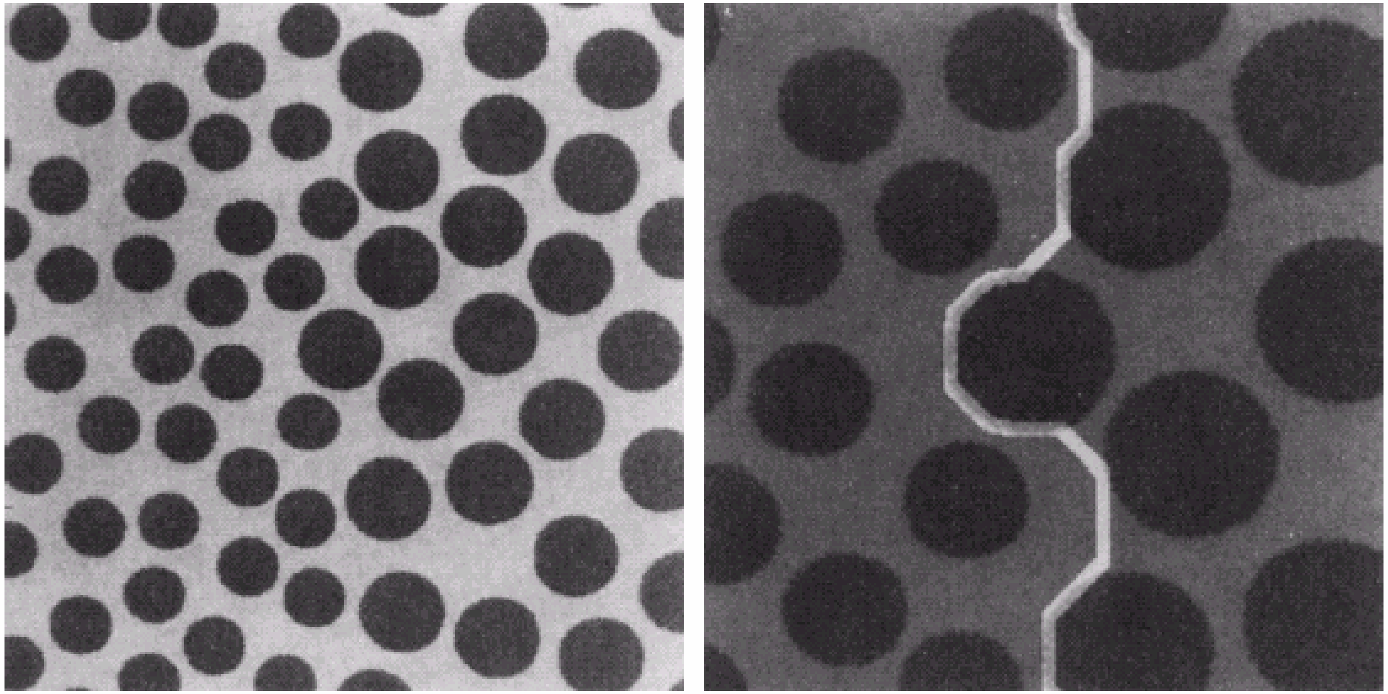
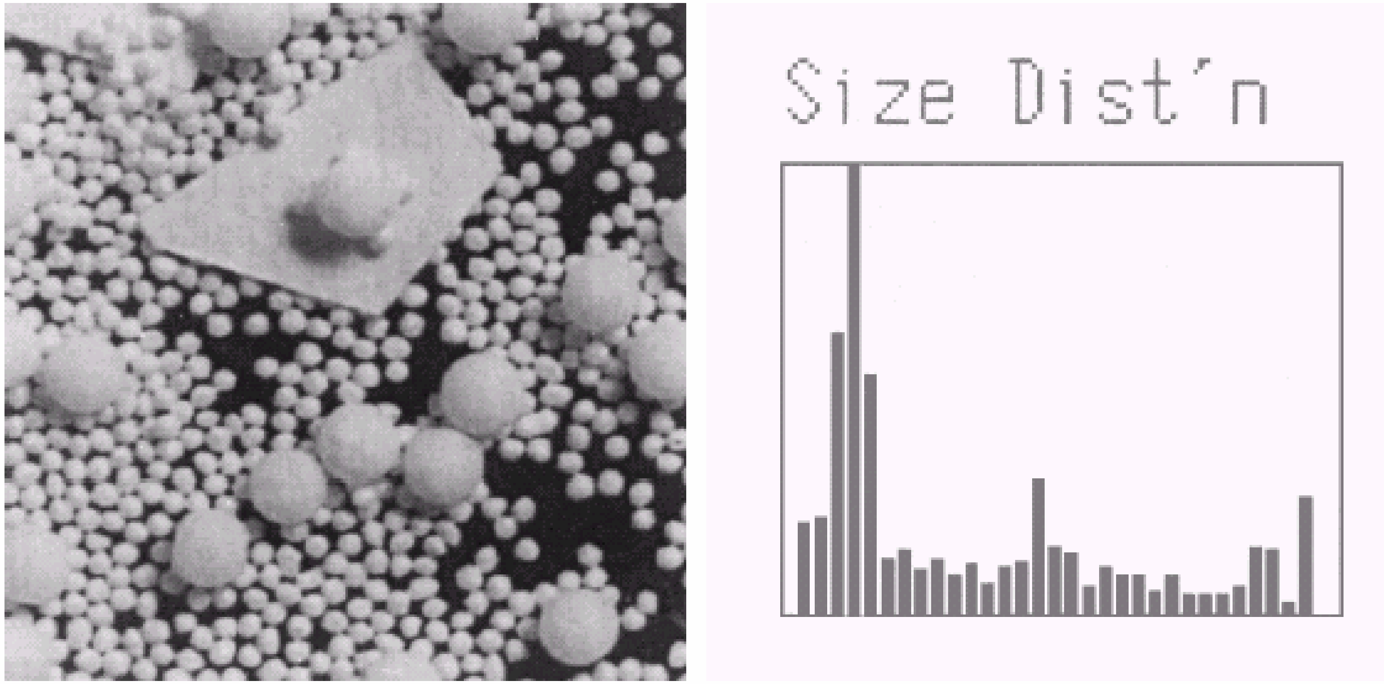


FIGURE 9.34 Result of performing a top-hat transformation on the image of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

a b

FIGURE 9.35
(a) Original image. (b) Image showing boundary between regions of different texture. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)





a b

FIGURE 9.36
(a) Original image consisting of overlapping particles; (b) size distribution. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)