Chapter 3 Image Enhancement in the Spatial Domain

Yinghua He

School of Computer Science and Technology Tianjin University



Image enhancement approaches

- Spatial domain
 - image plane itself
- Spatial domain methods
 - Based on direct manipulation of pixels in an image.
- Frequency domain methods
 - Based on modifying the Fourier transform of an image

Outline



• Background

- Some Basic Gray Level Transformations
- Histogram Processing
- Enhancement Using Arithmetic/Logic Operations
- Basics of Spatial Filtering
- Smoothing Spatial Filters
- Sharpening Spatial Filters
- Combining Spatial Enhancement Methods



- Spatial domain: the aggregate of pixels composing an image.
- Spatial domain methods: procedures that operate directly on these pixels.
- Spatial domain processes:

denoted by the expression:

$$g(x, y) = T[f(x, y)]$$

f(x,y):the input image

g(x,y):the processed image

T: an operator on f, defined over some neighborhood of (x,y).







- Gray-level transformation of the form
 - r: the gray level of f(x,y)
 - s: the gray level of g(x,y)
- Larger neighborhoods
 - masks: is a small 2-D array.

the mask coefficients determine the nature of the process, such as image sharpening.

 Masking processing
 Enhancement techniques based on this type of approach often are referred to as masking processing.





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- Three basic types of functions used frequently for image enhancement:
- Linear (negative and identity transformations)
- Logarithmic (log and inverse-log transformations)
- Power-law (nth power and nth root transformations)

Image negatives



• The negative of an image with gray levels in the range[0,L-1] is obtained by using the negative transformation in Fig. 3.3, which is given by the expression









a b

FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

Log Transformations



• The general form of the log transformation shown in Fig. 3.3 is $s = c \log(1 + r)$

where c is a constant, and it is assumed that $r \ge 0$.

• This transformation maps a narrow range of low gray-level values in the input image into a wider range of output levels.



a b FIGURE 3.5 (a) Fourier spectrum. (b) Result of applying the log transformation given in Eq. (3.2-2) with c = 1.



Power-Law Transformations

- Power-law transformations have the basic form $S = Cr^{\gamma}$
- Where c and γ are positive constants.











FIGURE 3.7

(a) Linear-wedge gray-scale image.
(b) Response of monitor to linear wedge.
(c) Gammacorrected wedge.
(d) Output of monitor.



 $s = r^{2.5}$ $s = r^{1/2.5}$



a b c d

FIGURE 3.8 (a) Magnetic

(a) Magnetic resonance (MR) image of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with c = 1 and $\gamma = 0.6, 0.4$, and 0.3, respectively. (Original image for this example courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)



a b c d

FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with c = 1 and $\gamma = 3.0, 4.0, \text{ and}$ 5.0, respectively. (Original image for this example courtesy of NASA.)



Piecewise-Linear Transformation Functions





a b c d FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)















FIGURE 3.13 An 8-bit fractal image. (A fractal is an image generated from mathematical expressions). (Courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA.)





FIGURE 3.14 The eight bit planes of the image in Fig. 3.13. The number at the bottom, right of each image identifies the bit plane.

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FIGURE 3.16 A gray-level transformation

transformation function that is both single valued and monotonically increasing.





FIGURE 3.17 (a) Images from Fig. 3.15. (b) Results of histogram equalization. (c) Corresponding histograms.







a b c

FIGURE 3.19 (a) Graphical interpretation of mapping from r_k to s_k via T(r). (b) Mapping of z_q to its corresponding value v_q via G(z). (c) Inverse mapping from s_k to its corresponding value of z_k .









FIGURE 3.20 (a) Image of the Mars moon Photos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)







FIGURE 3.21 (a) Transformation function for histogram equalization. (b) Histogramequalized image (note the washedout appearance). (c) Histogram of (b).



a c b d FIGURE 3.22 (a) Specified histogram. (b) Curve (1) is from Eq. (3.3-14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq. (3.3-17). (c) Enhanced image using mappings from curve (2). (d) Histogram of (c).







a b c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7×7 neighborhood about each pixel.



FIGURE 3.24 SEM

image of a tungsten filament and support, magnified approximately 130×. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).







a b c

FIGURE 3.25 (a) Image formed from all local means obtained from Fig. 3.24 using Eq. (3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq. (3.3-22). (c) Image formed from all multiplication constants used to produce the enhanced image shown in Fig. 3.26.




FIGURE 3.26

FIGURE 3.26 Enhanced SEM image. Compare with Fig. 3.24. Note in particular the enhanced area on the right side of the image.

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- Arithmetic/logic operations involving images are performed on a pixel-by-pixel basis between two or more images.
- Of the four arithmetic operations, subtraction and addition are the most useful for image enhancement.
- Masking sometimes is referred to as region of interest(ROI) processing.





- Image subtraction
- Image Averaging

Image subtraction



g(x, y) = f(x, y) - h(x, y)





a b

FIGURE 3.29

Enhancement by image subtraction. (a) Mask image. (b) An image (taken after injection of a contrast medium into the bloodstream) with mask subtracted out.



- Image subtraction
- Image Averaging



Consider a noisy image g(x,y) formed by the addition of noise η(x, y) to an original image f(x,y); that is

$$g(x, y) = f(x, y) + \eta(x, y)$$

$$\overline{g}(x, y) = \frac{1}{K} \sum_{i=1}^{K} g_i(x, y)$$



• Then it follows that $E\{\overline{g}(x, y)\} = f(x, y)$



$$\sigma_{\overline{g}(x,y)}^2 = \frac{1}{K} \sigma_{\eta(x,y)}^2$$







FIGURE 3.30 (a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging K = 8, 16, 64, and 128 noisy images. (Original image courtesy of NASA.)



a b FIGURE 3.31 (a) From top to bottom: Difference images between Fig. 3.30(a) and the four images in Figs. 3.30(c) through (f), respectively. (b) Corresponding histograms.



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- Some neighborhood operations work with the values of the image pixels in the neighborhood and the corresponding values of a subimage that gas the same dimensions as the neighborhood.
- The values in a filter subimage are referred to as coefficients, rather than pixels.
- For linear spatial filtering, the response is given by a sum of products of the filter coefficients and the corresponding image pixels in the area spanned by the filter mask.







 The result, R, of linear filtering with the filter mask at a point(x,y) in the image is

$$R = w(-1,-1)f(x-1, y-1) + w(-1,0)f(x-1, y) + \dots$$
$$+ w(0,0)f(x, y) + \dots + w(1,0)f(x+1, y) + w(1,1)f(x+1, y+1)$$

 In general, linear filtering of an image f of size M*N with a filter mask of size m*n is given by the expression:

$$g(x, y) = \sum_{s=-at=-a}^{a} \sum_{w=-a}^{b} w(s, t) f(x+s, y+t)$$



 When interest lies on the response, R, of an m*n mask at any point(x,y), it is common pratice to simplify the notation by using the following expression:

$$R = w_1 z_1 + w_2 z_2 + \dots + w_{mn} z_{mn} = \sum_{i=1}^{mn} w_i z_i$$



 For the 3*3 general mask shown below, the response at any point(x,y) in the image is given by

FIGURE 3.33

Another representation of a general 3×3 spatial filter mask.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9



- If the center of the mask moves any closer to the border, one or more rows or columns of the mask will be located outside the image plane.
- There are several ways to handle this situations.
 - To limit the excursions of the center of the mask to be at a distance no less than (n-1)/2 pixels from the border.
 - To filter all pixels only with the section of the mask that is fully contained in the image.
 - Padding the image by adding rows and columns of 0's, or padding by replicating rows and columns.

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- Smoothing filters are used for blurring and for noise reduction.
 - Blurring is used in preprocessing steps, such as removal of small details from an image prior to object extraction, and bridging of small gaps in lines or curves.
 - Noise reduction can be accomplished by blurring with a linear filter and also by non-linear filtering.



• Smoothing Linear Filter

Order-Statistics Filter



- The output of a smoothing, linear spatial filter is simply the average of the pixels contained in the neighborhood of the filter mask.
- This filters sometimes are called averaging filters. They also referred to a lowpass filter.



$$R = \frac{1}{9} \sum_{i=1}^{9} z_i$$

 The general implementation for filtering an M*N image with a weighted averaging filter of size m*n is given by the expression

$$g(x, y) = \frac{\sum_{s=-at=-b}^{a} \sum_{w=-b}^{b} w(s, t) f(x + s, y + t)}{\sum_{s=-at=-b}^{a} \sum_{w=-b}^{b} w(s, t)}$$





a b

FIGURE 3.34 Two 3×3 smoothing (averaging) filter masks. The constant multipli er in front of each mask is equal to the sum of the values of its coefficients, as is required to compute an average.





- c d
- e f

a b **FIGURE 3.35** (a) Original image, of size 500×500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes n = 3, 5, 9, 15, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50×120 pixels.





a b c

FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15×15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)



• Smoothing Linear Filter

• Order-Statistics Filter



- Order-statistics filters are nonlinear spatial filters whose response is based on ordering (rankng) the pixels contained in the image area encompassed by the filter, and then replacing the value of the center pixel with the value determined by the ranking result.
- The best-known example in this category is the filter, which, as its name implies, replaces the value of a pixel by the median of the gray levels in the neighborhood of that pixel.





a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

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- The principal objective of sharpening is to highlight fine detail in an image or to enhance detail that has been blurred, either in error or as a natural effect of a particular method of image acquisition.
 - Averaging is analogous to integration;
 - Sharpening could be accomplished by spatial differentiation.



• Foundation

- Use of Second Derivatives for Enhancement-The Laplacian
- Use of First Derivatives for Enhancement-The Gradient



- We require that any definition we use for a first derivative.
 - Must be zero in flat segments(areas of constant gray-level values);
 - Must be nonzero at the onset of a gray-level step or ramp;
 - Must be nonzero along ramps.



• The definition of a second derivative

- Must be zero in flat areas;
- Must be nonzero at the onset and end of a graylevel step or ramp;
- Must be zero along ramps of constant slope.



 A basic definition of the first-order derivative of a one-dimensional function f(x) is the difference.

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

• We define a second-order derivative as the difference.

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$




FIGURE 3.36

Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.



- Comparing the response between first- and secondorder derivatives, we arrive at the following conclusions.
 - First-order derivatives generally produce thicker edges in an image.
 - Second-order derivatives have a stronger response to fine detail, such as thin lines and isolated points.
 - First-order derivatives generally have a stronger response to a gray level step.
 - Second-order derivatives produce a double response at step changes in gray level.



 Second-order derivatives that, for similar changes in gray-level values in an image, their response is stranger to a line than to a step, and to a point than to a line.



- Foundation
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- Use of First Derivatives for Enhancement-The Gradient



- We are interested in isotropic(同向性) filters, whose response is independent of the direction of the discontinuities in the image to which the filter is applied.
- Isotropic filters are rotation invariant.



 The simplest isotropic derivative operator is the Laplacian, which, for a function (image)f(x,y) of two variables is defined as

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

• Because derivatives of any order are linear operations, the Laplacian is a linear operator.



• We use the following notation for the partial second-order derivative in the x-direction:

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

• And, similarly in the y-direction, as

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$



• The digital implementation of the twodimensional Laplacian is obtained by summing these two components.

$$\nabla^2 f = \left[f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) \right] - 4f(x, y)$$

0	1	0	1	1	1
1	-4	1	1	-8	1
0	1	0	1	1	1
0	-1	0	-1	-1	-1
-1	4	-1	-1	8	-1
0	-1	0	-1	-1	-1



a b c d

FIGURE 3.39 (a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.



- Because the Laplacian is a derivative operator, its use highlights gray-level discontinuities in an image and deemphasizes regions with slowly varying gray levels.
- The basic way in which we use the Laplacian for image enhancement is as follows:

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) \\ f(x, y) + \nabla^2 f(x, y) \end{cases}$$

if the center coefficient of the Laplacian mask is negative if the center coefficient of the Laplacian mask is positive

a b c d

FIGURE 3.40 (a) Image of the North Pole of the moon. (b) Laplacianfiltered image. (c) Laplacian image scaled for display purposes. (d) Image enhanced by using Eq. (3.7-5). (Original image courtesy of NASA.)



Simplifications



g(x, y) = f(x, y) - [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] + 4f(x, y)= 5f(x, y) - [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)]



a b c d e FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Unsharp masking and highyboost filtering



- A process used for many years in the publishing industry to sharpen images consists of subtracting a blurred version of an image from the image itself. This process, called unsharp masking, is expressed as $f_s(x, y) = f(x, y) \overline{f}(x, y)$
- Where f_s(x, y)denotes the sharpened image obtained by unsharp masking and f
 (x, y) is a blurred version of f(x, y).



 A slight further generalization of unsharp masking is called high-boost filtering. A highboost filtered image, f_{hb}, is defined at any point (x,y) as

$$f_{hb}(x, y) = Af(x, y) - \overline{f}(x, y)$$

where A>=1

• The equation may be written as

$$f_{hb}(x, y) = (A - 1)f(x, y) - f_s(x, y)$$

0	-1	0	-1	-1	-1
-1	A + 4	-1	-1	A + 8	-1
0	-1	0	-1	-1	-1



a b

FIGURE 3.42 The high-boost filtering technique can be implemented with either one of these masks, with $A \ge 1$.



• If we select to use the Laplacian, the above equation becomes

$$f_{hb} = \begin{cases} Af(x, y) - \nabla^2 f(x, y) \\ Af(x, y) + \nabla^2 f(x, y) \end{cases}$$

if the center coefficient of the Laplacian mask is negative if the center coefficient of the Laplacian mask is positive

• When A=1, high-boost filtering becomes "standard" Laplacian sharpening. a b c d

FIGURE 3.43 (a) Same as Fig. 3.41(c), but darker. (a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using A = 0. (c) Laplacian enhanced image using the mask in Fig. 3.42(b) with A = 1. (d) Same as (c), but using A = 1.7.





 For function f(x,y), the gradient of f at coordinates (x,y) is defined as the twodimensional column vector

$$\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

• The magnitude of this vector is given by $\nabla f = mag(\nabla f) = \left[G_x^2 + G_y^2\right]^{\frac{1}{2}} = \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{\frac{1}{2}}$



 In order to reduce the computational burden, it is common practice to approximate the magnitude of the gradient by using absolute values instead of squares and square roots:

$$\nabla f \approx \left| G_{x} \right| + \left| G_{y} \right|$$



- The simplest approximations to a first-order derivative that satisfy the conditions stated are $G_x = (z_8 z_5)$ and $G_y = (z_6 z_5)$.
- The other definitions proposed by Roberts in the early development of digital image processing use cross differences:

$$G_x = (z_9 - z_5)$$
 and $G_y = (z_8 - z_6)$



• We compute the gradient as

or

$$\nabla f = \left[(z_9 - z_5)^2 + (z_8 - z_6)^2 \right]^{\frac{1}{2}}$$

$$\nabla f \approx \left| z_9 - z_5 \right| + \left| z_8 - z_6 \right|$$

• An approximation using absolute values, still at point z_5 , but using a 3*3 mask, is

$$\nabla f \approx \left| (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3) \right| \\ + \left| (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7) \right|$$







FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

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a b c d

FIGURE 3.46

(a) Image of
whole body bone
scan.
(b) Laplacian of
(a). (c) Sharpened
image obtained
by adding (a) and
(b). (d) Sobel of
(a).





FIGURE 3.46 (Continued) (e) Sobel image smoothed with a 5×5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

