

Dynamic Programming Assignments

April 24, 2014

Problem 1: Weighted Staged Directed Graph: WSDG

In a weighted staged directed graph G , we want to search the shortest path from start node s to the terminal node e .

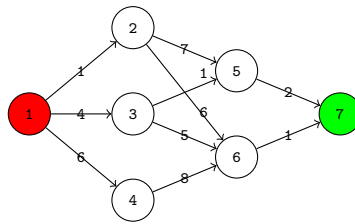


Figure 1: A weighted staged directed graph

1. Defined that $g(i)$ is the shortest distance from start node s to the node i in a WSDG. Write out the recursive equation.
2. Write out the pseudo codes to calculate the shortest distance $g(i)$ using your recursive equation.

Problem 2: Knapsack problem

The knapsack problem is a problem in combinatorial optimization: Given a set of items, each with a mass and a value, determine the number of each

item to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible.

Let there be n items, z_1 to z_n where z_i has a value p_i and weight w_i . x_i is the number of copies of the item z_i . The maximum weight that we can carry in the bag is c . It is common to assume that all values and weights are nonnegative.

The task of knapsack problem is to determine the values of x_i so that

$$\max \sum_{i=1}^n x_i * p_i \quad (1)$$

$$\text{Subjects to: } \sum_{i=1}^n x_i * w_i \leq c \quad (2)$$

1. 0/1 knapsack problem in which x_i must be zero or one. Show that the problem holds the principle of optimization(optimal substructure).
2. 0/1/2 knapsack problem in which x_i must be zero, one or two. Define the optimal value function of the problem and derive corresponding recursive equation.